STATICS I

EQUILIBRIUM

Statics is the study of forces acting on objects that do not move. Such objects are said to be in static equilibrium. .

We have defined force to be something that causes a **displacement** when applied to a body, so static equilibrium requires that there is no resultant force acting on (applied to) the object.

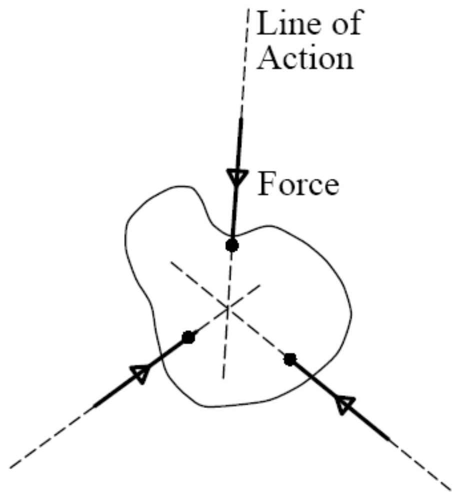
This is to say that there must be no resultant force acting on the object, or that the **vector sum** of all the forces acting must be zero. This leads to the first important principle:

STATICS EXAMPLE 1

ABCDEF is a regular hexagon. Forces represented by AB, FA, BC and 2DE act on a particle. Prove that the particle is in equilibrium.

Condition for Static Equilibrium (restricted version)

For a body to be in equilibrium under the action of a number of concurrent forces, the resultant of these forces must be zero.



Notes

1. **concurrency** means that the lines of action of all the forces pass through the same point:
2. A more general condition that can be applied to situations in which the forces are not concurrent will be given later in the module.

Special Case of Equilibrium with 3 Forces

Situations in which objects are in equilibrium under the action of 3 forces occur frequently, and special techniques have been developed to deal with them. The following principle can be very useful in solving statics problems, specifically in determining the magnitude of one or more unknown forces.

Principle of Concurrency

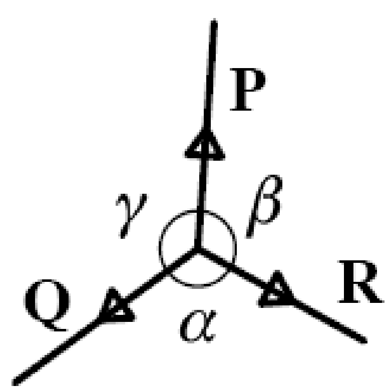
If an object is in equilibrium under the action of three non–parallel forces, then these three forces must be concurrent.

Notes

1. This principle is a corollary of the condition for condition for static equilibrium .
2. It is sufficient to know that there are 3 forces acting; that these are co-planar; and that the object is in static equilibrium
3. The principle can be used in situations where there appear to be more than 3 forces acting, providing that by taking intersections of sets of forces acting at particular points, 3 non–parallel forces can be obtained.

Lami's Theorem

If a body is in equilibrium under the action of 3 concurrent forces **P**, **Q** and **R** as shown in the diagram:



then:

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

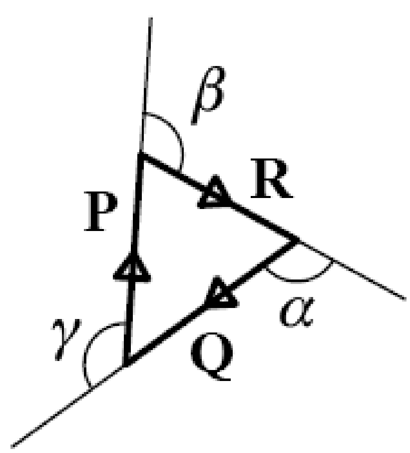
where, for simplicity, the line *P*, for example, has been used to represent the vector **P**

.

Note The angle is to the line of action of the force vector **P** and so on.

Proof

Lami's theorem is derived by applying the **Sine Rule** to the lines of action of forces. Since **P**, **Q** and **R** are in equilibrium, the 3 force vectors can be arranged to form a triangle. If this is done, the angles shown above become the **exterior** angles as can be seen from the figure:



By the Sine Rule:

$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

Using the **Identity** $\sin(180 - \theta) = \sin\theta$ this is readily converted into the expression stated above.

STATICS PROBLEMS

All statics problems essentially involve finding all the forces acting on a body, and using the equations of static equilibrium to show that the body is in static equilibrium, or to find reactions.

The critical part is **defining** all the forces acting on the body. As it is easy to miss some out, it is very useful to have a checklist of the types of force that may be encountered.

Types of Force

1. Applied Forces

Fairly obvious. May be **actual forces** or they may be ties. For example, wires, strings, ropes etc attached to objects can only apply a tensile force acting away from the object. The direction of the force applied is the **direction of** the wire, string etc.

2. Weight

Unless it is stated that the object is **weightless** , one of the forces acting will be the object's **weight**, acting through the centre of a particle (usually). The magnitude of the weight is the mass of the object multiplied by the acceleration of a body when falling freely under gravity, that is *mg* .

For bodies (that are not particles) the weight vector can be taken to act (be applied to) the centre of mass of the body.
The calculation of the position of the centre of mass of arbitrary bodies is beyond the scope of this course. We shall confine ourselves to bodies for which the centre of mass is easily found by, for example, a sphere.

3. Reactions
These are forces acting as a result of contacts (usually) with other objects. Reactions are easy to forget as they are not always stated explicitly. For frictionless surfaces, the reaction force always acts normally (at 90^o) to the surfaces in contact. When two objects are in contact, the reaction force on one of them points towards the other object. We shall look at specific examples of contact later on.

STATICS EXAMPLE 2

A particle of weight *W* is suspended inside a smooth hemispherical bowl by a string attached to the rim of the bowl. Find the reaction on the particle and the tension in the string if the length of the string is equal to the radius of the bowl.

ENERGY

ENERGY = WORK = Force x Displacement

$$U = \sum F \bullet \Delta$$

"Displacement" is at the point of application and in the <u>same direction</u> as the Force i.e. a "Dot Product"

ENERGY MINIMISATION

The motion of bodies will tend towards "Minimum Energy"
"Equilibrium" is a minimum energy state
Minimum Energy:

$$\frac{\partial U}{\partial \Delta} = 0$$

$$\therefore \frac{\partial U}{\partial \Delta} = \sum F = 0$$

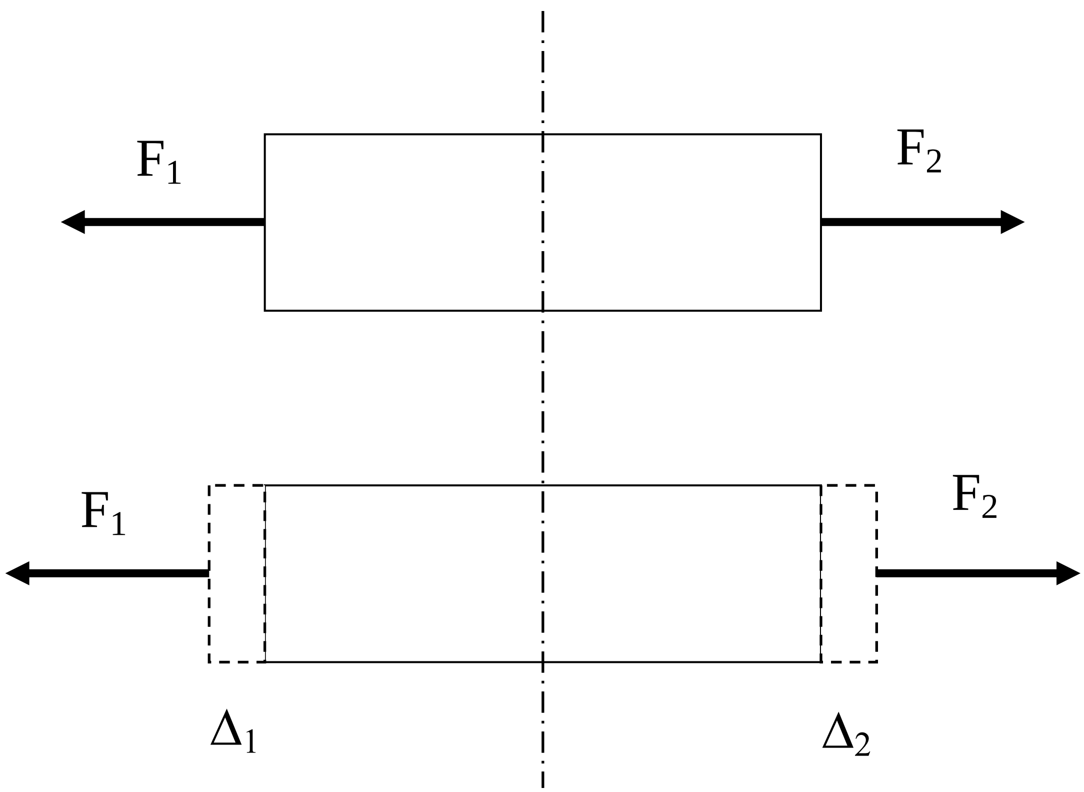
$$\therefore \sum F = 0$$

Equilibrium: the sum of external forces = 0

Equilibrium: Virtual Work =0

EQUILIBRIUM

Translation Equilibrium



Change in Total Potential Energy = 0

Virtual Work = 0

$$\Delta U_V = -F_1\Delta_1 + F_2\Delta_2 = 0$$

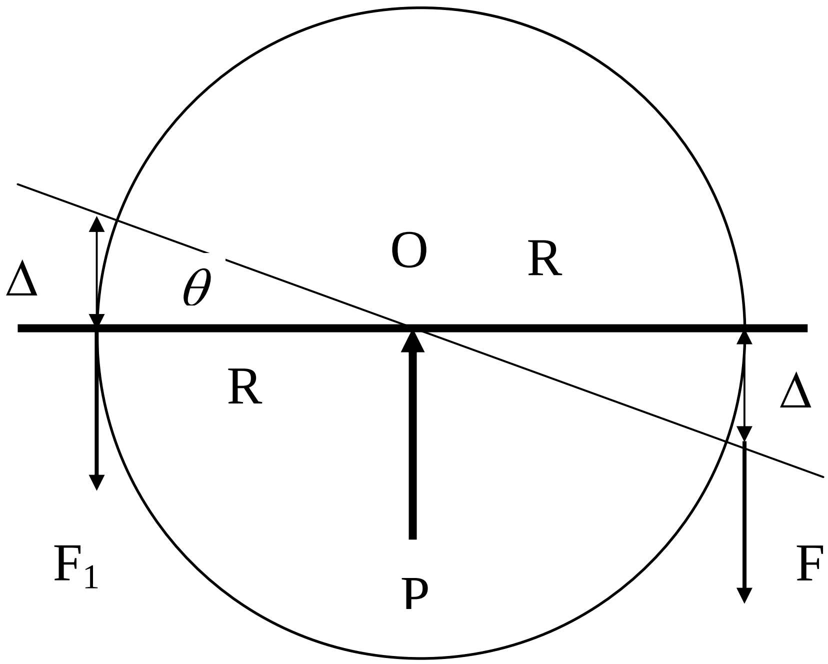
$$\text{if } \Delta_1 = \Delta_2 = \Delta \text{ then} \div \text{through by } \Delta$$

$$\therefore \sum \mathrm{F_H} = -F_1 + F_2 = 0$$

Horizontal Equilibrium: Virtual Work = 0
∴ Summation of horizontal forces equals zero

EQUILIBRIUM

Rotational Equilibrium



Virtual Work: Rotation about 'O'

$$\Delta U_V = F_2\Delta_2 - F_1\Delta_1 + P(0) = 0$$

but $\Delta_1 = -RSin\theta \quad \Delta_2 = +RSin\theta$

$$\therefore U_V = F_2 RSin\theta - F_1 RSin\theta = 0$$

$\div$ thro by $Sin\theta$

Let "Moment" $= M = FR$

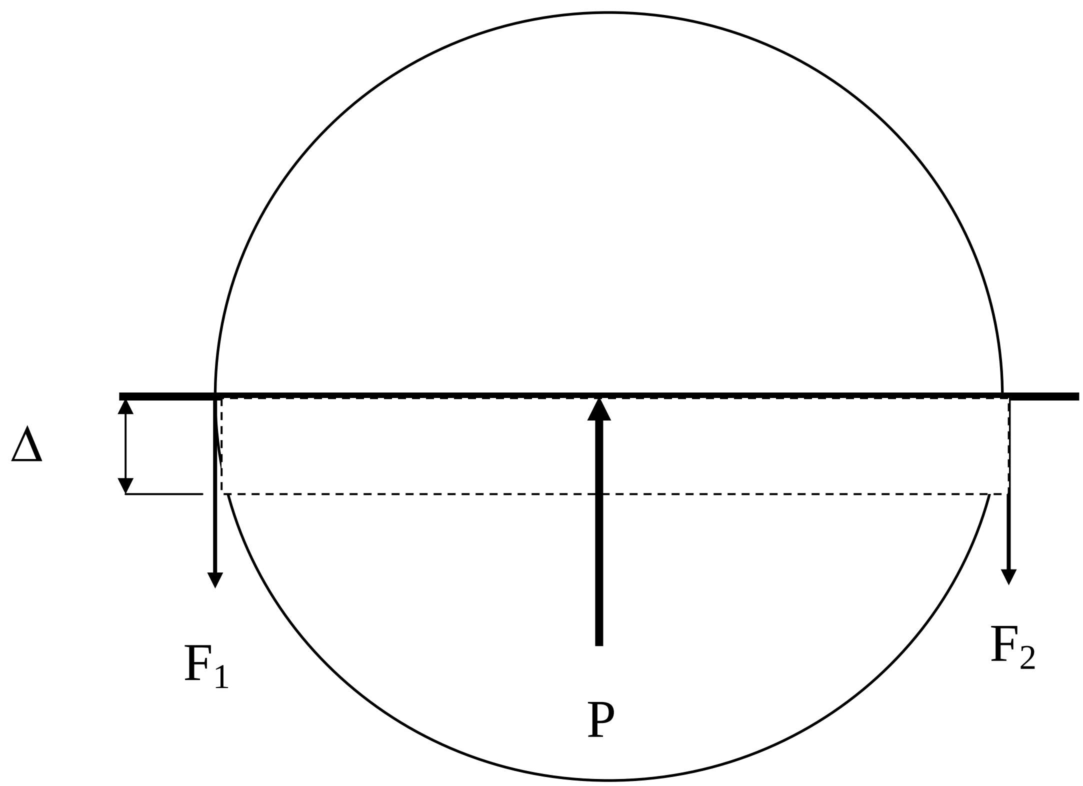
$$\therefore U_V = \sum M_O = M_2 - M_1 = 0$$

$$\therefore \quad M_2 = M_1$$

Rot'l Equilibrium: Virtual Work = 0

EQUILIBRIUM

Rotational Equilibrium



Virtual Work

$$\Delta U_V = 0 = F_1\Delta + F_2\Delta - P\Delta = 0$$

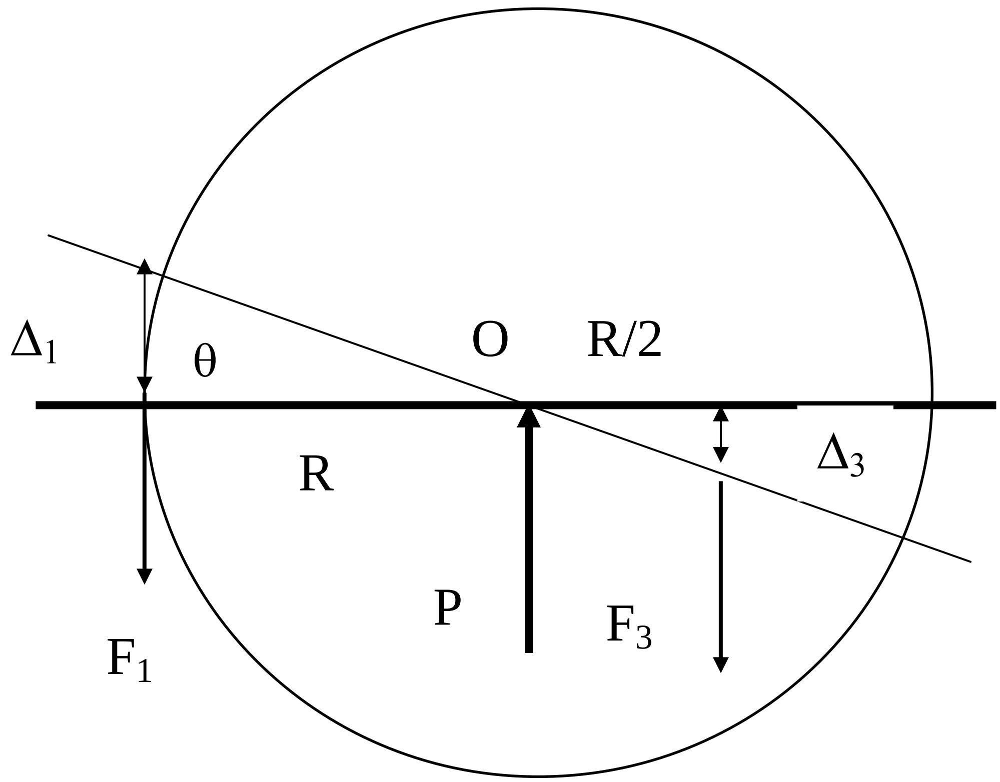
$$\div \text{ thro by } \Delta$$

$$\therefore \quad P = F_1 + F_2 \quad \therefore \sum F_V = 0$$

Vertical Equilibrium:
Summation of Vertical Forces = 0

EQUILIBRIUM

Rotational Equilibrium



Virtual Work

$$U_V = -F_1\Delta_1 + F_3\Delta_3 + P(0) = 0$$

but $\Delta_1 = RSin\theta$ and $\Delta_2 = (R/2)Sin\theta$

$\div$ thro by $Sin\theta$

$$\therefore\ \Delta U_V = \sum M_O = -F_1R + F_3\,R/2 = 0$$

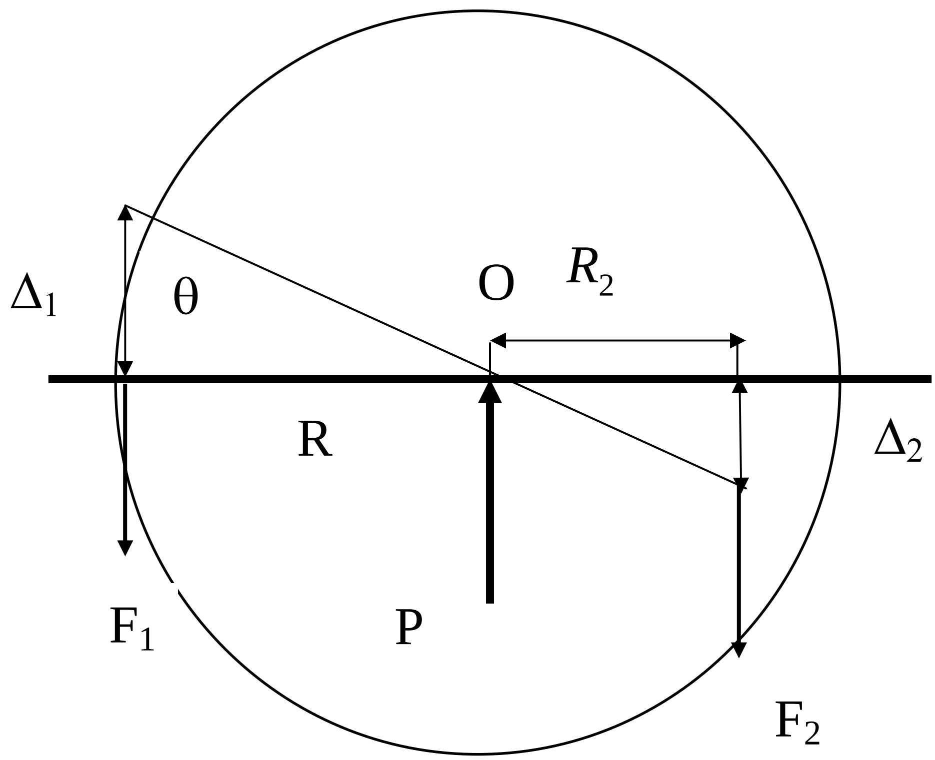
$$\therefore\ F_1R = F_3\,R/2$$

$$\therefore\ F_3 = 2F_1$$

Rotational Equilibrium:
Virtual Work = 0

EQUILIBRIUM

Rotational Equilibrium



Virtual Work

$$\Delta U_V = -F_1\Delta_1 + F_2\Delta_2 + P(0) = 0$$

but $\Delta_1 = RSin\theta$ and $\Delta_2 = R_2Sin\theta$

$$\therefore\ U_V = -F_1RSin\theta + F_2R_2Sin\theta = 0$$

$$\therefore\ U_V = \sum M_O = -M_1 + M_2 = 0$$

Where $M_1 = F_1R$ and $M_2 = F_2R_2$

EQUILIBRIUM

Rotational Equilibrium

- **The “MOMENT” of a force is: the FORCE multiplied by the “MOMENT ARM”.**

- **The MOMENT ARM is the distance from the point under consideration to the intersection with the orthographic projection of the “Line of Action” of the force.**

EQUILIBRIUM:
VIRTUAL WORK = 0

EQUILIBRIUM

Equilibrium:
The sum of external forces = 0

“Virtual Work”

U_V = Force x “Virtual” Displacement

$$U_V = \sum F.\Delta$$

$$\text{but } \sum F = 0$$

$$\therefore \quad U_V = 0$$

$$\text{Equilibrium} : \text{Sum of external forces} = 0$$

$$\sum F_V = \sum F_H = \sum M = 0$$

STATICS EXAMPLE 3

A particle of weight 10 N is attached to two strings. Find the tension in the strings if they make angles of 30° and 45° to the vertical.

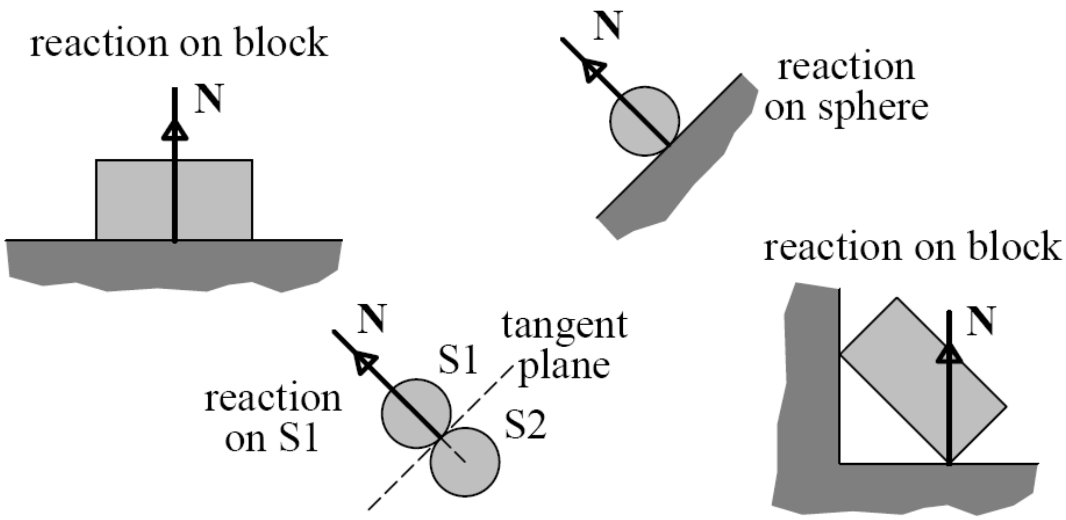
Note Quite often, the easiest way to use the condition of static equilibrium in problems is to equate both (2D) **Components of the forces** to zero.

Contact Between Bodies:

Objects which are in contact exert **contact** forces on each other.

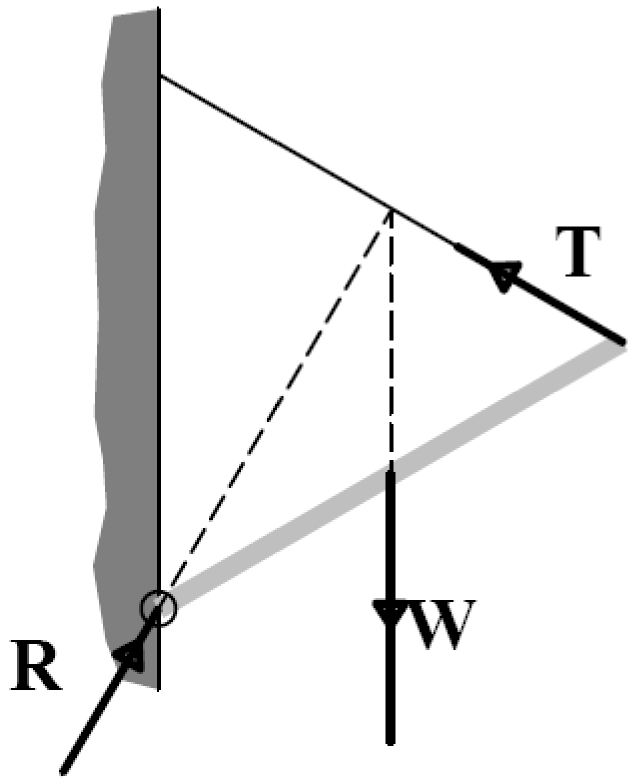
Frictionless Contact

Since, by definition, there can be no component of force acting tangentially between the two surfaces, the contact force for (frictionless) surfaces always acts **normal (at 90^o)** to the **surfaces** This is referred to as the normal reaction N.



Frictionless Hinges and Pivots

Frictionless hinges and pivots provide no restraint to rotation of the parts, but they do prevent any translation of the pivot point. The line of action of the reaction force is therefore determined simply by the geometry of the mechanism, as in frictionless contact, but also by concurrency.



The direction of the **reaction** **R** at the support shown in the figure is determined by the **Principle of Concurrency**, using the known **directions** of the weight vector **W** and the tension force in the rope **T**.

STATICS EXAMPLE 4

A weight of 20 N rests on a smooth surface inclined at an angle of 25° to the horizontal and is prevented from slipping by a string inclined at an angle of 45° to the surface. Find the tension in the string.

Note In this case it was only necessary to resolve in one direction since the value of **N** was not required. The choice of a direction normal to the slope was intentional because it is the direction of **N** .

STATICS EXAMPLE 5

A particle of weight 15 N is hanging at the end of a string. It is displaced sideways by a horizontal force until the string makes an angle of 30° to the vertical. Find the magnitude of the force and the tension in the string.

STATICS EXAMPLE 6

A uniform bar AB of length L and weight W is hinged at A to a horizontal surface and rests on a smooth cylinder of radius $3L/8$ lying on the surface a distance $3L/4$ from point A with its axis aligned with the axis of rotation of the hinge. Find the reaction at the hinge.