# KINEMATICS I

Kinematics is the study of MOTION. Generally, we are not concerned with the forces that cause the motion, that is the subject of the next unit.
In this unit we shall confine our attention to motion in straight lines (rectilinear motion), and deal with more complicated examples later (see Unit : Kinematics II). By considering just linear motion, we do not have to bother about the directions of displacement, velocity and acceleration vectors, except to specify which way along the line they are pointing (that is by using the component values, having established a overall (general) direction).

To start with, we shall consider linear motion with constant velocity.
Velocity has been defined to be the **change in displacement** per

unit time, so for a constant velocity, the change in displacement is
the same in any time interval.

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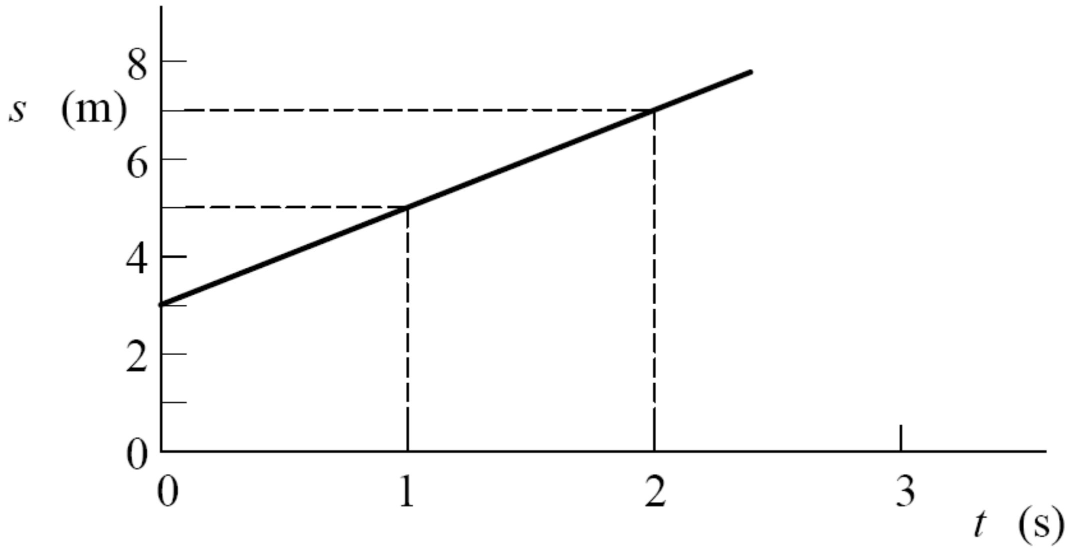
KINEMATICS EXAMPLE 1

A particle moves in a straight line with uniform velocity so that at time $t = 0$, its displacement from point O is 3 m, and at time $t = 1$ s, its displacement is 5 m. Find its velocity and its displacement at time $t = 2$ s.

**Displacement–Time Graphs**

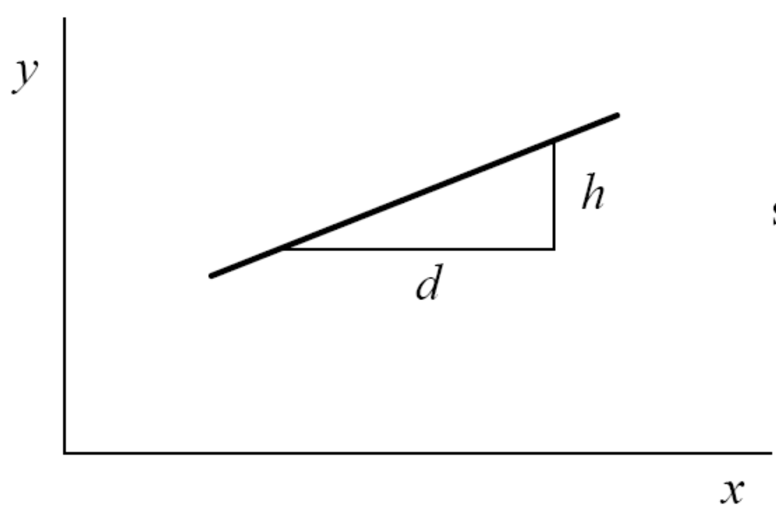
These are a useful way of representing **motion** in a straight line. For example, the displacement–time graph for Example 1 would be:




## Slope of Displacement–Time Graphs

From basic geometry, the slope of a straight line is:

$$\frac{\text{change in vertical position}}{\text{change in horizontal position}}$$




$$\text{slope} = \frac{h}{d}$$

But for a displacement–time graph:

change in vertical position = change in displacement

and

change in horizontal position = change in time

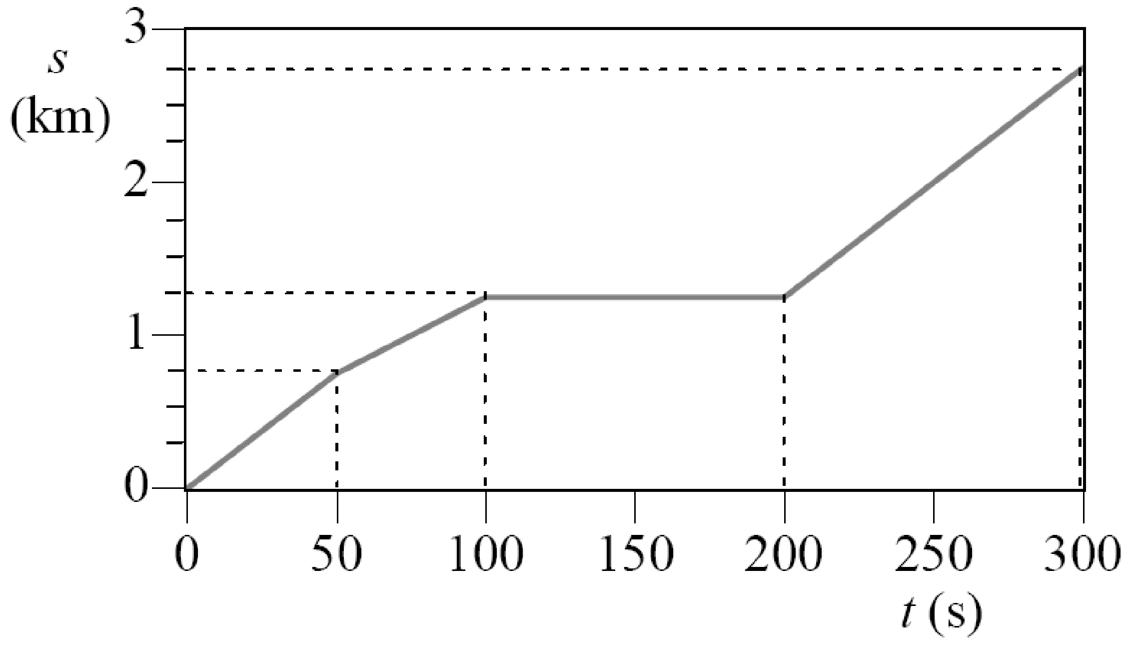
so, for uniform velocity, the slope of a displacement–time graph is the AVERAGE VELOCITY.

**Motion with Different Uniform Velocities in Successive Time Intervals**

The previous ideas can be extended to encompass motion in which the velocity is constant for a while, then changes and remains constant for another time interval and so on.

At this point we assume the velocity can change instantaneously from one value to another. It cannot, of course, but we will soon move on to more realistic situations.

KINEMATICS EXAMPLE 2
The following displacement–time graph represents the motion of a car along a straight road. Find the velocity at time 1 minute, and the time taken to travel a distance of 2 km.



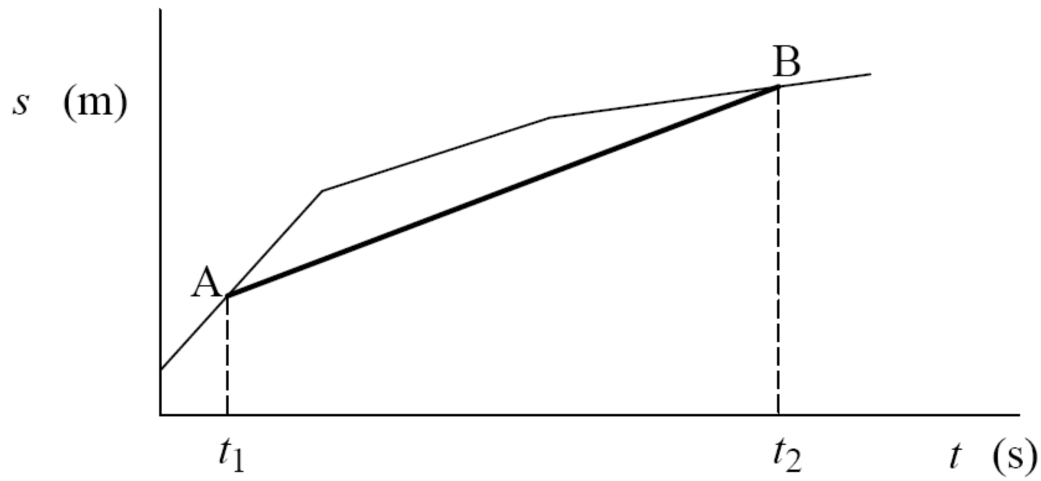
## Average Velocity and Average Speed

When velocity changes, it is useful to define an avearge velocity, that is the uniform velocity which gives the same overall displacement as the **average** one in a given time interval. Formally:

$$\text{Average Velocity} = \frac{\text{change in displacement}}{\text{corresponding time interval}}$$

***Note*** This is the definition given earlier for velocity – the addition of the word average simply emphasises that it is applicable to theoverall motion, whether the velocity is varying or not.

It can easily be seen that the **average velocity** during an interval of time is the slope of the straight line joining the points A & B on the displacement–time graph:




The average velocity in the above figure between times *t*1 and *t*2 is the slope of the line AB.

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***Note*** It is very important so specify a **a time interval** when using average velocity: the concept of **average velocity** is meaningless unless it is known over which interval of time the average velocity is calculated.

Average speed can be defined in a similar way, bearing in mind that speed, unlike velocity, takes no account of the direction of motion:

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{corresponding time interval}}$$

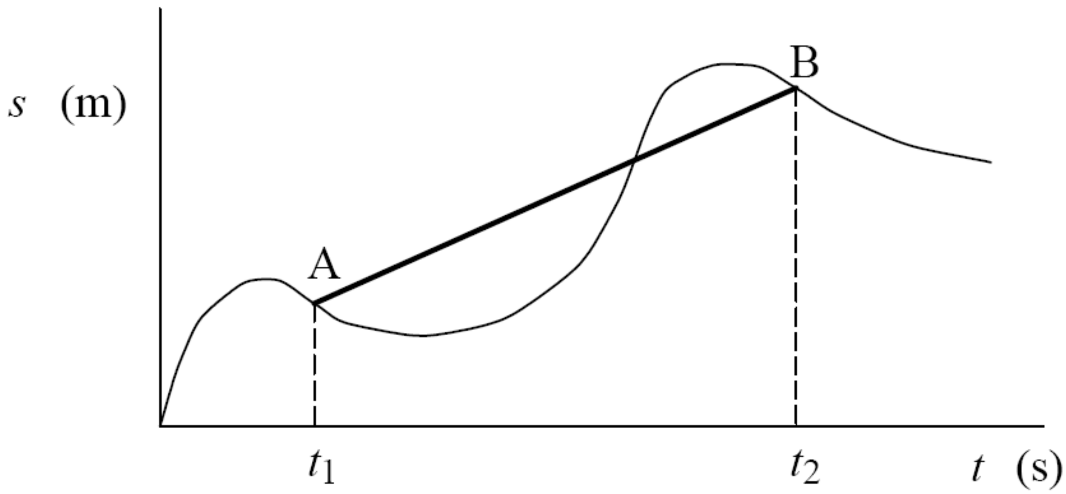
Average speed and average velocity are often the same in linear motion, but **NOT** when the direction of motion changes, as the next example shows.

KINEMATICS EXAMPLE 3

A hiker walks up a hill at constant speed taking 10 minutes to cover a distance of 800 m. She rests for 2 minutes and then walks down at constant speed reaching a point halfway down in 3 minutes. Draw a displacement–time graph for the journey and find the average speed and the average velocity for the journey between the start and the halfway point on the way down.

**Motion with Varying Velocity**

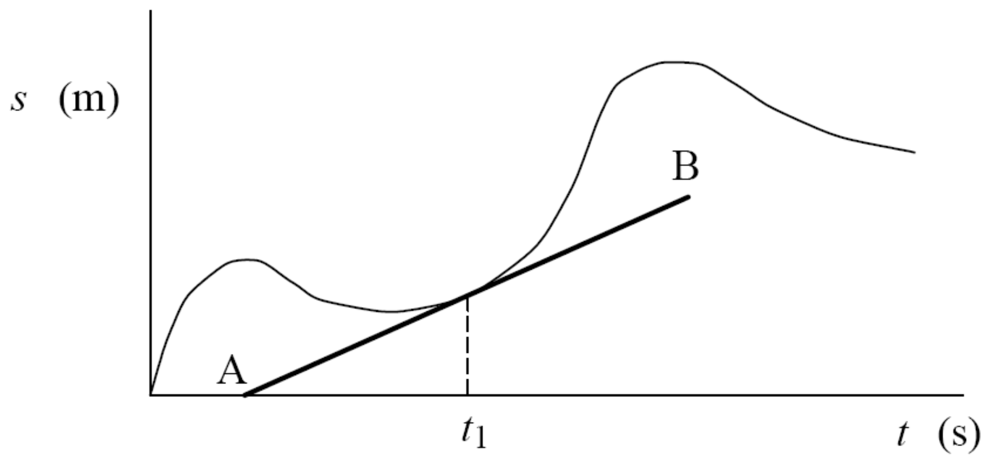
The definitions of **average velocity** and average speed apply equally well to motion in which the velocity is changing all the time:




The average velocity between times *t*1 and *t*2 is still the slope of the line AB.

In addition, for varying velocity, it is possible to define the velocity at a given time by extension of the property of the uniform–velocity displacement–time graph:

Instantaneous Velocity = slope of the tangent to the displacement–time graph at that instant




The instantaneous velocity at time $t_1$ is the slope of the line AB.

There are basically three ways of determining the velocity at an instant:

1. Calculus, that is from the function describing the displacement (if known) using differentiation. .

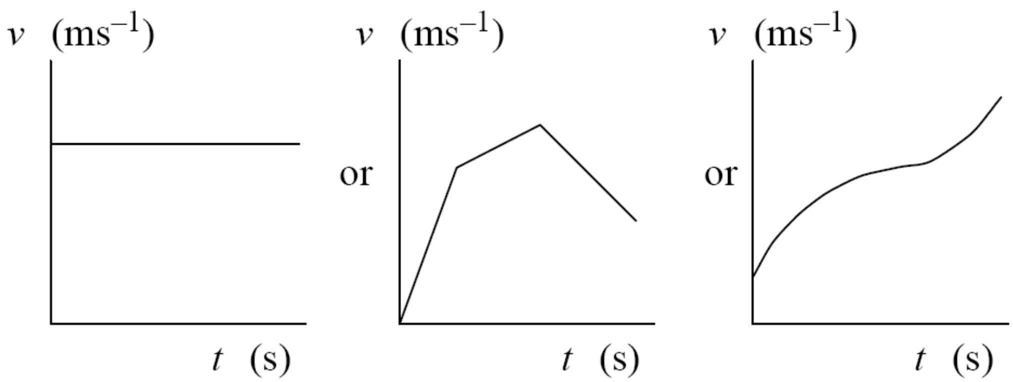
2. Geometry, that is by plotting the displacement–time graph and drawing the Tangent. .
3. **By estimation**, that is by finding the average velocity over a time interval using the tabulated data or function.

Approach 1. is beyond the scope of this course, but is straightforward once calculus has been mastered.
Approach 2. is obvious and need not be dealt with further.
The following class example illustrates the third approach.

KINEMATICS EXAMPLE 4
A particle moves in a straight line so that at any time $t$, its displacement $s$ from a fixed point O on the line is $t2–5t$. Sketch the displacement–time graph for the interval $t = 0$ to $t = 6$ s. Estimate the velocity at $t = 2$ s.

## Velocity–Time Graphs

Just as previously we plotted the displacement of an object as a function of time, it is often useful to plot a graph showing how the velocity changes with time, for example:

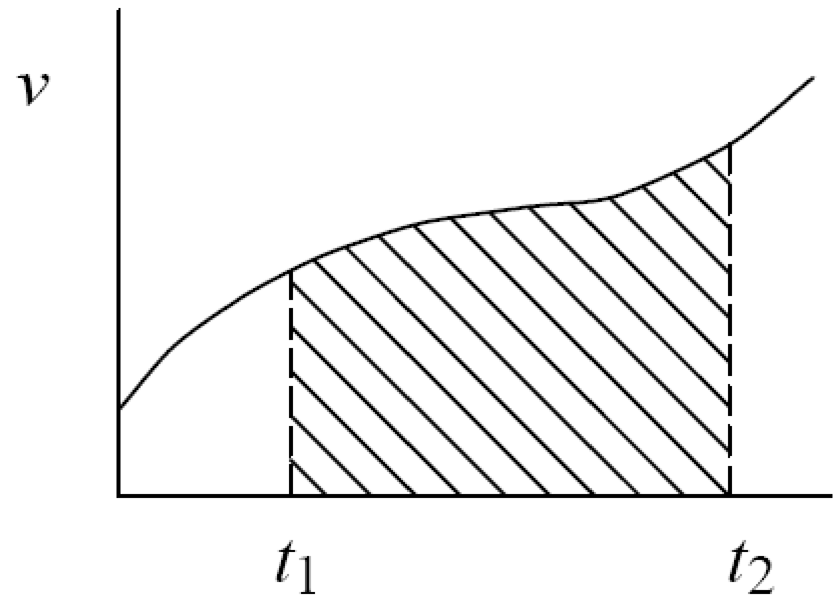



## Properties of Velocity–Time Graphs

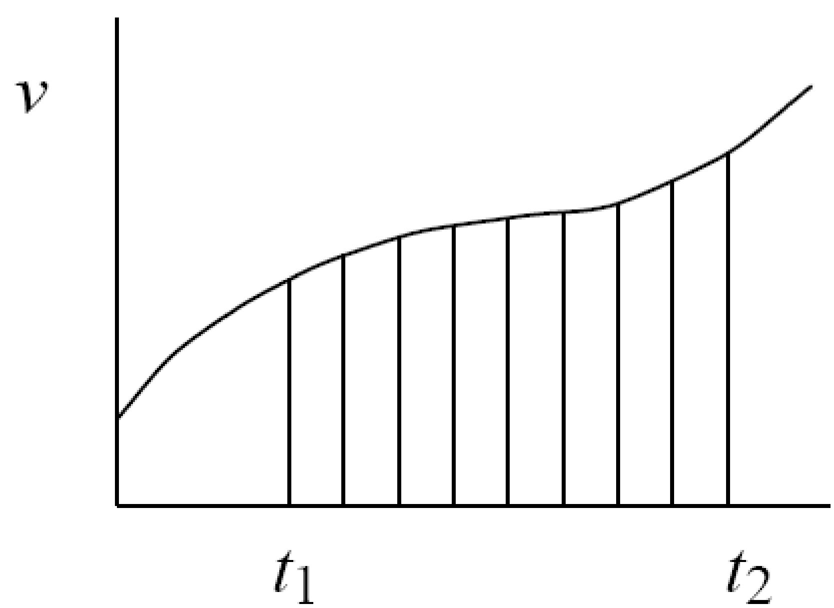
1. the slope of the graph at any time is the acceleration at that instant (so a straight–line graph indicates constant acceleration)
2. the area under the graph lying between verticals drawn at two different times is the velocity between those two times.

**Notes**

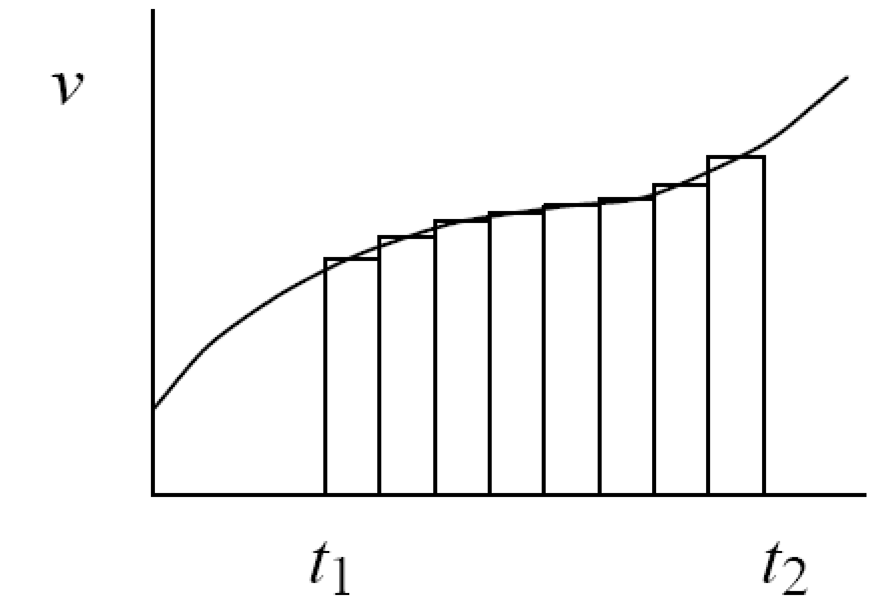
1. The first property is quite general, whether the graph at the instant is a straight line or not. If the graph is **curved**, find the Tangent. Beware corners!
2. The first property can be derived in the same way the slope of the displacement–time graph was shown to be the velocity.
3. A rigorous proof of the second property requires calculus,
but in broad terms it goes as follows:




Divide the time interval up into a number of smaller time intervals:



If the new time steps are small enough, the velocity can be assumed to be **constant** during each small time step:




The area of each **rectangle** is a constant velocity times a small change in time, that is the distance during that time step. Therefore, the area under the curve, which is approximately the **SUM** of the areas of all the thin rectangles, will be the **distance traveled** between times $t1$ and $t2$ .

4. Areas are formed from the horizontal time axis are equal time steps.                 .

KINEMATICS EXAMPLE 5

A particle moving in a straight line is uniformly accelerated from rest by an acceleration of 4 ms–2 for 4 seconds and then subjected to a constant retardation of 8 ms–2 for 4 seconds. Draw the velocity–time graph for the interval 0 to 8 seconds and find the velocity at time 5 seconds, the total distance travelled at the end of the interval and the average velocity for the whole motion.

***Note*** a Retardation  of 5 ms–2 (for example) is the same as an acceleration of –5 ms–2.

# Equations of Motion for Linear Motion with Constant Acceleration

Examples of linear motion with constant acceleration occur frequently. When determining the displacement, velocity or acceleration from the displacement–time or velocity–time graphs, the same variables are encountered over and over again, but with different values.
It makes sense therefore to remove the graphical stage altogether, and to just use the algebraic expressions for the motion.

There are five standard equations of linear motion.

$$s = \frac{u+v}{2}t \qquad (1)$$

$$s = ut + \frac{1}{2}at^2 \qquad (2)$$

$$s = vt - \frac{1}{2}at^2 \qquad (3)$$

$$v = u + at \qquad (4)$$

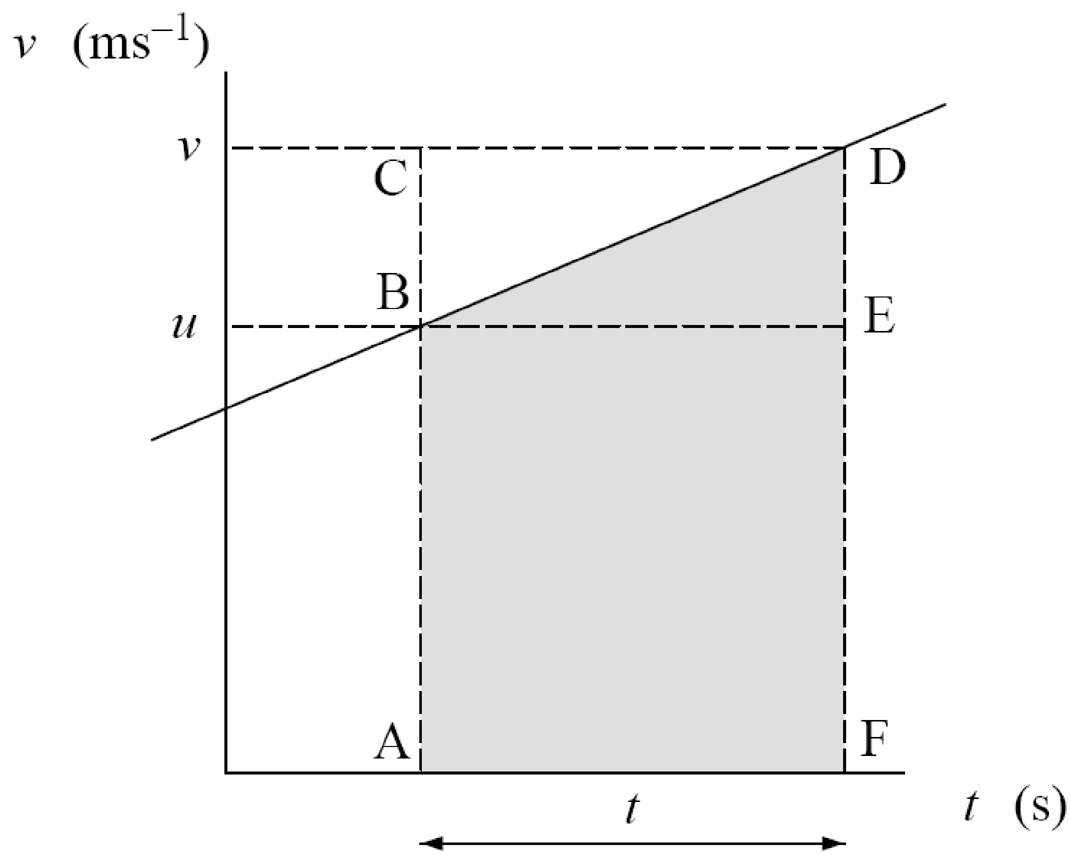
$$v^2 = u^2 + 2as \qquad (5)$$

**Notes**

1. *t* is a variable (not a time instant)
*u* is the initial velocity at the start of the interval
*V* is the **final velocity** at the end of the interval
*a* is the constant **acceleration**
*s* is the displacement during that interval.
2. The equations of motion crop up so often it is very worthwhile memorising them, though it is possible to derive them when needed if all else fails.

# Derivation of Equations of Linear Motion

This is fairly straightforward, and uses a velocity time graph for the motion:




*t* (s)

***Note*** the acceleration is constant, so the graph is a straight line.

# *s* is the area of the **Trapezium** ABDF.

1.

$$\text{area of trapezium} = \frac{1}{2}(\text{AB} + \text{DF})\ t$$

$$s = \frac{1}{2}(u + v)\ t$$

2.

$$\text{area of trapezium} = \text{area ABEF} + \text{area BDE}$$

$$= ut + \frac{1}{2}\text{DE}\ t$$

but DE is the ____________________ in time $t$, which is equal to $at$ so:

$$s = ut + \frac{1}{2}at^2$$

3.

$$\text{area of trapezium} = \text{area ACDF} - \text{area BCD}$$

$$= vt - \frac{1}{2}\text{BC}\ t$$

$$s = vt - \frac{1}{2}at^2$$

4. The acceleration is the ________________ BD, so:

$$a = \frac{v-u}{t}$$

$$v-u = at$$

$$v = u + at$$

5. Combine equations 1 and 4:

$$s = \frac{1}{2}(u+v)\ t$$

$$t = \frac{v-u}{a}$$

so:

$$s = \frac{(u+v)(v-u)}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

## Problem Solving

1. There are only 2 Independent equations – the other 3 can always be derived from these.

5 equations are stated for convenience in problem solving.

2. There also 5 variables (*u*, *v*, *a*, *t*, *s* ) so if any 3 of these are stated, the other **two** can be evaluated.

3. Which of the 5 equations you use depends on the information given in the problem.

4. Problems in which velocity changes from one constant value to another in a time interval can
still be solved by repeated application of the standard equations of motion.

KINEMATICS EXAMPLE 6

A particle moving in a straight line is started from rest by a uniform acceleration. After 5 seconds, it is moving with a velocity of 10 ms–1 and is then subject to a constant deceleration. If the particle comes to rest after covering a total distance of 75m find the distance travalled while the velocity is increasing, and the acceleration during this stage, the value of the constant deceleration and the total time taken for the motion.

## Vertical Motion due to Gravity

This is a special case of linear motion with constant acceleration, in which the acceleration is implicitly the value of $g$ .

## Notes

1. The vertical direction may be chosen to be retardation (acceleration is $-g$ ) or acceleration (acceleration is $+g$ ).
The choice is a matter of convenience.
2. The value of $g$ will always be given in questions related to this module, but in general problem solving it can take values of 10, 9.8 or 9.81 ms–2 depending upon the desired **accuracy** of the solution. $g$ actually varies with height above the Earths

surface, and the Latitude. At sea level at the Equator, it's value is approximately 9.78 ms–2, while at the Poles it
is approximately equal to 9.83 ms–2. The value of 9.81 is applicable to the British Isles and northern Europe at sea level.

3. Unless stated otherwise, ignore air resistance.

4. 'At rest' means there is no initial velocity. 'travelling at', '**moving with**' etc means there is some initial velocity.

KINEMATICS EXAMPLE 7

A stone is dropped from the top of a tall building and hits the ground with a velocity of 42 ms–1. Find the height of the building and the time taken for the descent ($g$ = 9.81 ms–2).