DYNAMICS II

MOMENTUM

This is the second important characteristic of the motion of a body (the first was **Inertia**, see Dynamics I).
The momentum of a body is the product of its mass and its
velocity (a vector) :

$$\mathbf{p} = m\mathbf{v}$$

The units of momentum are kg m s–1 .
Note since **v** is a vector, so is momentum.
The significance of kinetic energy was its ability to do work, i.e. the force acting on a body multiplied by the distance moved during the application of this force.
The significance of momentum is its relationship to the
Force multiplied by the Time . This is given the name IMPULSE. .

Formally then, the Impulse **J** of a constant force **F** acting on a body for a time *t* is:

$\mathbf{J} = \mathbf{F}\,t$

Impulse is measured in N s .
If no force acts on a body, its velocity is constant (a consequence of **Newton's First law**) so its **acceleration** must also be constant, assuming the mass is constant. Therefore, for the momentum of a body to change, a force must act upon it for a given time i.e. an **Impulse** must be present.
The relationship between impulse and change in momentum is easily derived from Newton's Second Law, and the standard equations of motion.

Let a constant force **F** act on a body of mass *m* for a time *t* ,changing its velocity from **u** to **v** :

Newton II: $$F = ma$$

Eqn. of motion: $$v = u + at$$

or: $$a = \frac{v - u}{t}$$

Hence:

$$F = m\frac{v - u}{t}$$

or: $$Ft = mv - mu$$

$$impulse = change\ in\ momentum$$

DYNAMICS EXAMPLE 10

A ball of mass 200 grams is thrown towards a wall so that it strikes the wall normally with a speed of 6 ms^{-1}. If the ball bounces at right angles away from the wall with a speed of 4ms^{-1}, what impulse does the wall exert on the ball?

Notes

1. Since impulse equals change in momentum, impulse and momentum must have the same units. Hence kg m s–1

and N s are equivalent ways of expressing the same quantity. Which you use depends on the context of the question.

$$F(N) = m(kg)a(ms^{-2})$$

$$\therefore N = kgms^{-2}$$

$$\therefore Ns = kgms^{-1}$$

2. The expression for impulse is for a constant force. It is possible to define impulse when the force is **varying**, but this involves calculus.

3. The relationship between **Impulse** and **momentum** was derived from Newton's second law of motion. In fact, a more general way of expressing this law is:

force equals rate of change of momentum

i.e.

$$\mathbf{F} = \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t}$$

If m is **constant**, this simplifies to the $F = ma$ law we are familiar with. However, the more general law also allows for the possibility that **momentum** may change because mass changes.

4. If **F** and t are considered constant, then we have derived no special benefit from the introduction of the concept of **Impulse**. However, it is in situations in which **F** and t may not be known separately, but for which the change in momentum may be calculated that the concept is particularly relevant. This is generally the case for Impact problems, such as the previous example.

5. Even when problems can be solved without introducing the ideas of Impulse or momentum, these quantities are often useful in providing a concise description of the mechanical behaviour.

DYNAMICS EXAMPLE 11

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In passing at right angles through a wooden block of thickness 5 centimetres, the speed of a bullet of mass 50 grams is reduced from 120 m s^{-1} to 80 m s^{-1}. Find the average force exerted on the bullet while it is in contact with the block and the time taken for it to pass through.

Conservation of Momentum

Consider a system of bodies in motion. Each component (body) will have **mass** and **velocity**, and hence **momentum.** Allow the individual bodies to **collide** in some way. The nature of the collision is not important, and may be action at a distance (for example gravity) or direct force.

When two bodies interact, Newton's Third Law states that the force exerted on one must be equal to the force exerted on the other. But the duration of the interaction is the same for both, so the impulses acting on them must also be **equal**.

Thus during an interaction, the mmentum of the two bodies may change, but any **change** in momentum of one will be exactly compensated by an **equal change** in momentum of the other. That is, the **momentum** of the two bodies is CONSERVED.

But this will be true for the interaction between any pair of bodies in the system, and so the **MOMENTUM** of the system is CONSERVED, providing no **external force** acts on it.

This principle can be generalised slightly to consider the Conservation of Linear of momentum.

Principle of Conservation of Linear Momentum

The component of the momentum of a body in a given direction is conserved providing no external force acts on the system.

DYNAMICS EXAMPLE 12

A truck of mass 400 kg runs at a speed of 2 m s^{-1} into a stationary truck. They become coupled together and move on with a speed of 0.8 m s^{-1}. What is the mass of the second truck?

Problem Solving

The most important consideration is deciding what your system consists of and making sure that there is no external force acting in a particular direction.

For example, **two spheres** colliding on a

Frictionless surface may constitute a suitable system because no external force is acting on them , and momentum is conserved in any direction within the plane. However, a ball hitting a **wall** does not satisfy these requirements because the **wall** exerts **an external force** on the ball. It is no good including the **wall** in the system, because the **ground** exerts an external force on the **wall**, and so on.

Direct Elastic Impact

Problems in which two or more bodies travelling along the same **linear path** collide, one against the other, occur frequently in classical mechanics so it is worth examining these situations in more detail.

Consider two particles of mass *m* travelling with

The same speed along the same linear path **towards** each other, i.e. one has velocity +**u** , the other –**u** .

After **impact**, let the velocities be **v**1 and **v**2 respectively. Then, by the **conservation of Linear mometum**:

$$mu - mu = mv_1 + mv_2$$

i.e. $v_1 = -v_2$

so the velocities after impact are once again **equal and opposite**.

However, we do not know their value!

Suppose the collision is **perfectly elastic**. By definition (you are required to be familiar with this definition) , this means that during the impact any **kinetic energy** that may be converted to **elastic potential energy** as the particles **distort** is completely converted back again to **kinetic energy** as they recover their original shape. Thus the **kinetic energy** before and after is **the same** (no energy is converted to **heat or sound** etc):

$$\text{KE before impact} = \frac{1}{2}mu^2 + \frac{1}{2}mu^2$$

$$\text{KE after impact} = \frac{1}{2}mv_1^2 + \frac{1}{2}m(-v_1)^2$$

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therefore:

$$mu^2 = mv_1^2$$

and:

$$v_1 = \pm u$$

Clearly the first particle will have velocity –**u** and the second particle will have velocity +**u** (so that they **move apart**).

By contrast, suppose that **all** the initial **kinetic energy** is converted to **heat or sound** during the impact (we might call this situation *perfectly inelastic)*. Then:

$$\text{KE after impact} = mv_1^2 = 0 \quad => \quad v_1 = 0$$

that is, both particles **come to rest** and in contact.

Clearly there will be an infinite number of other possibilities between these two extremes, where only some of the **kinetic energy** is ‘lost’.

Note **conservation of momentum** gives us no way to distinguish between all these possibilities: it only tells us that $\mathbf{v}_1 = -\mathbf{v}_2$.

We need some factor, dependent upon the **energetics** of the collision, that will allow us to uniquely determine $\mathbf{v}_1$ and $\mathbf{v}_2$.

This is provided by the **coefficient of restitution** *e* defined as follows:
coefficient of restitution

$$e = \frac{\text{separation speed}}{\text{approach speed}}$$

For example, in the previous situation, for the **perfectly elastic impact**:

$$\begin{aligned} e &= \frac{v_2 - v_1}{u - (-u)} \\ &= \frac{u - (-u)}{2u} \\ &= \frac{2u}{2u} = 1 \end{aligned}$$

For **perfectly inelastic impact**:

$$e = \frac{0}{2u} = 0$$

In general:

$$0 \leq e \leq 1$$

DYNAMICS EXAMPLE 13

A sphere A of mass 0.1 kg is moving with a speed of 5 m s^{-1} when it collides directly with a stationary sphere B. If A is brought to rest by the impact and e = 0.5, find the mass of B and its speed just after impact.

Problem Solving

Multiple collisions can be dealt with by considering collision between **successive pairs** of bodies.