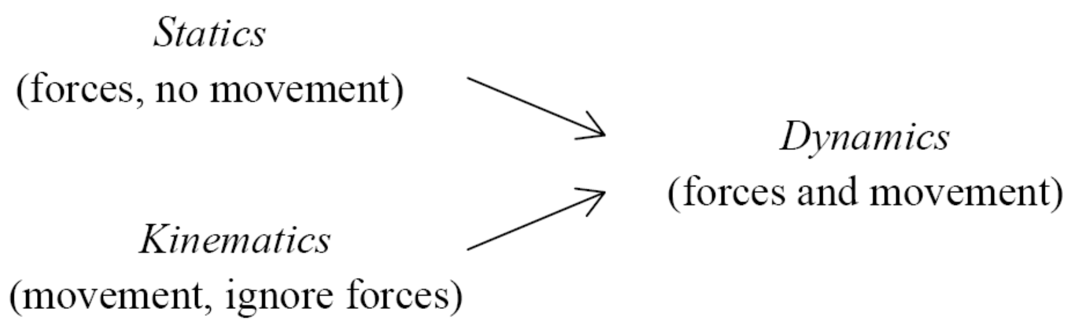
DYNAMICS I

Dynamics is the study of **Forces** acting on moving bodies. There is therefore a logical progression:



The link between forces and **motion** is provided by Newton's laws of motion.

NEWTON'S LAWS OF MOTION

1. A body remains in a state of rest or continues to move with constant velocity unless acted on by an external force.
2. The acceleration of a body of constant mass is proportional to the external force applied to that body.

3. When body A exerts a force on body B, B exerts an equal and opposite force on A.

Isaac Newton (1642–1727) Lucasian Professor of Mathematics at Cambridge from 1669.

Notes

1. Newton's laws underpin classical mechanics, which is why the subject is often called Newtonian Mechanics.
2. In the first law of motion, 'continues' is the same as saying 'moving with constant velocity or remaining at rest'.
3. The statements of the three laws of motion given here use modern terminology (Newton expressed them differently, and in Latin!) Alternative, but equivalent forms are possible.
4. Each of the three laws has its own **significance** and leads to important results.

5. Newton's three laws of motion should be memorised. You can't derive them at his stage.

Newton's First Law of Motion

The significance of the first law of motion is that it leads to a generalisation of the state of equilibrium. .

In Unit 1: Statics I, the idea of **equilibrium** was introduced, that is the state in which bodies exist when they do not move. It was stated there that a condition for this was that there should be no resultant (overall) **force** acting on the body.

Newton's first law states that this is a necessary condition for a body not to move, but not a sufficient one, for a body may move with constant velocity even when no forces are

acting.
Thus although the condition for equilibrium stated earlier is quite correct, the concept of equilibrium needs to be extended to cover:

(a) static equilibrium – no movement
(b) dynamic equilibrium – constant velocity

The thing that is common to both of these is, of course, that the body is subject to forces, and this will take us later to Newton's second law of motion.

Note you can, if you wish, just consider static equilibrium to be a special case of dynamic equilibrium, that is one in which the (constant) velocity happens to be zero.

The idea that objects can be in equilibrium even when they are moving had a revolutionary effect on the study of mechanics.

The concept of static equilibrium has a very long history, and for a long time people had understood that if a body is **stationary** the forces somehow had to be in ' **balance**,' but the idea that objects with no forces, or zero resultant force, acting on them can carry on indefinitely with the same velocity seems to contradict common sense.

For example, a stone thrown through the air does not travel in a straight line, it traces out a parabola (and slows down); a ball rolling along a flat horizontal surface does not carry on for ever, even if it meets no obstruction – eventually it will

stop due to friction, and as a further apparent contradiction, and one which was of great interest to scientists at the time of Newton, planets do not travel in

circular orbits, but follow ellipses around the sun.
The explanation for these apparent contradictions is that in all these cases, there are **forces** acting which had not been consisdered, in the case of the stone and the ball, air resistance and gravity in the case of the stone and the planets. The notion of gravitational attraction was, of course, another of Newton’s contributions to science.

DYNAMICS EXAMPLE 1

A packing case is dragged in a straight line across a rough level floor at a constant speed by means of a rope inclined at an angle of 30° to the horizontal. If the frictional force between the case and the floor resisting the motion is 100 N, what is the tension in the rope?

Newton's Second Law of Motion

The significance of the second law of motion is that it provides a link between the **forces** acting on a body and the **motion** of that body.

Mathematically, the second law states that:

$$\mathbf{a} = k\,\mathbf{f}$$

for some constant of proportionality k .

Note This is a vector equation. One consequence of this is that **a** is in the direction as **f** .

Experimentally, by measuring the acceleration of bodies of different mass for the same **force**, it is found that the acceleration is proportional to the mass, that is:

$$\mathbf{a} = k' \frac{1}{m} \mathbf{f}$$

for some different proportionality factor *k*' .

The value of *k*' will depend upon the units chosen to measure **a** , *m* and **f** . Conventionally, units of **f** are chosen so that when *m* is measured in kg and **a** is measured in ms^{-2}, then the value of *K*' is noe and can be dropped. Thus:

$$\mathbf{f} = m\mathbf{a}$$

which is the most usual way of expressing Newton's second law of motion. The **unit of force** derived in this way is the Newton (abbreviation N) that we have assumed all along.

Formally then:

Force is the action which, when applied to a object of mass ‘m’ , causes a change in acceleration of ‘m’. .

A **Newton** is the force which, when applied to an object of mass of 1 kg causes an acceleration of 1 ms^{-2}.

Note The statement of the second law given here assumes the body has **constant mass**. A more general form, which can be applied to situations in which the mass might change (a rocket for example) will be looked at later.

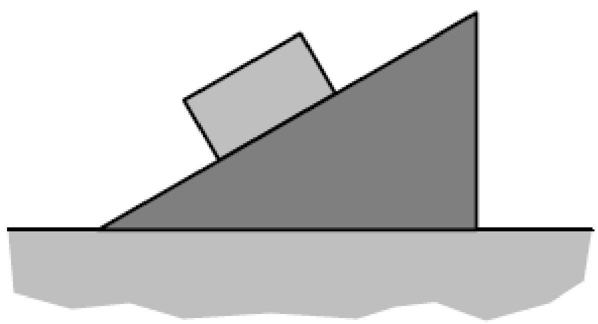
DYNAMICS EXAMPLE 2

A particle of mass 50 grams is acted on by a force
of 2**i** + **j** N. What is its acceleration?

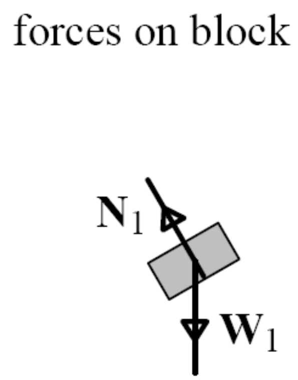
Newton's Third Law of Motion

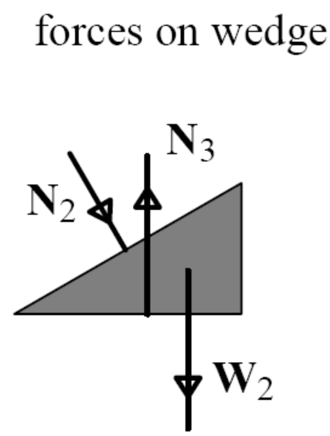
The significance of Newton's third law is that it allows complex systems to be considered as an aggregation of simpler ones.

For example, a block is in contact with the inclined plane of a smooth wedge, which is itself resting on a smooth surface:



Considering "Free Bodies"

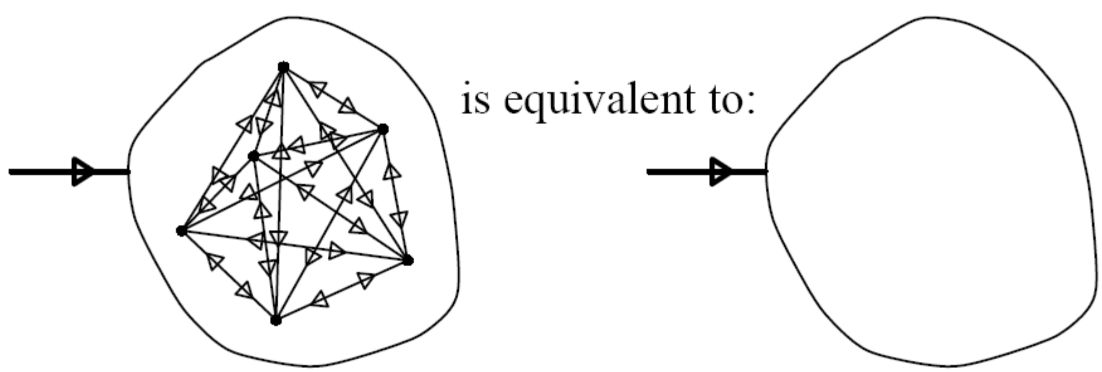




Thus both parts of the system will be subject to an acceleration, and the equations for these can be derived **the motions**. When solving them, however, Newton's third law can be used to show that **N**1 = – **N**2, thereby **simplifying** the two sets of equations.

Another important implication of Newton's third law is that the relative motion of a system can often be ignored if it is only the behaviour of the system as a whole that is required.

For example, an object is made of an extremely large number of atoms or molecules, each of which exerts a force on all the others (mainly electromagnetic). But if we wish to know how the object moves under an external force, this atomic or molecular structure can be ignored, because the forces between any pair of atomic particles are **in equilibrium** and **sum to zero**:



Note Newton's third law is true whether the bodies are in motion or not.

DYNAMICS EXAMPLE 3

Find the force exerted by a crate of mass 50 kg on the floor of a lift when the lift and crate are ascending with an acceleration of 2 ms^{-2}.

DYNAMICS EXAMPLE 4

A block of mass 500 grams is in contact with the smooth sloping face of a wedge of mass 1 kg which is resting on a smooth horizontal surface. Find the acceleration of the wedge if its sloping face makes an angle of 45° with the horizontal.

RELATED MOTION

Contact is not the only way in which one object can exert a force on another. One important category of mechanical problems involves bodies connected by strings, ropes, wires, cables etc, possibly with the inclusion of pulleys to alter the magnitude and direction of 'pull'.

Problem Solving

1. The method of solution is the familiar one of breaking the complicated problem down into a collection of simpler ones, (Free Bodies) and using Newton's second law to obtain the equations governing their motion.
2. The difficulty is finding relationships between the forces acting on the various parts of the system, and between the **motions** (specifically the relative motions) of these parts.
3. Providing all pulleys are **connected**, the **force** in all parts of the connecting wire (etc) will be **same**. The direction of the force vector acting on the various

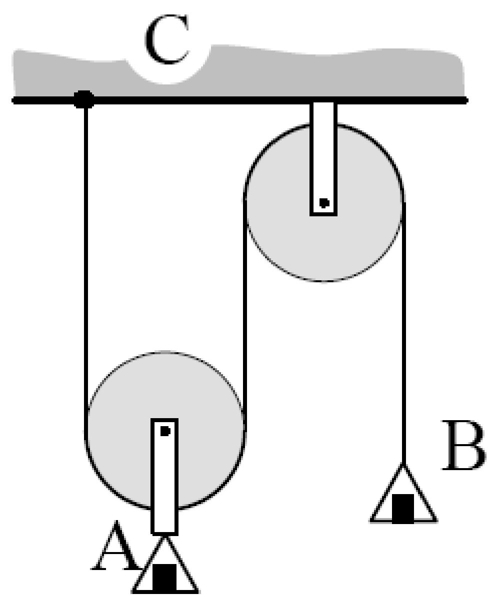
parts of the system will depend of course upon the direction of the wire that is attached to them. Assume pulleys are frictionless unless stated otherwise.

4. The velocity and acceleration of different parts of the system will be related by the **properties** of the wire and pulleys and has to be determined **for each part of the system**

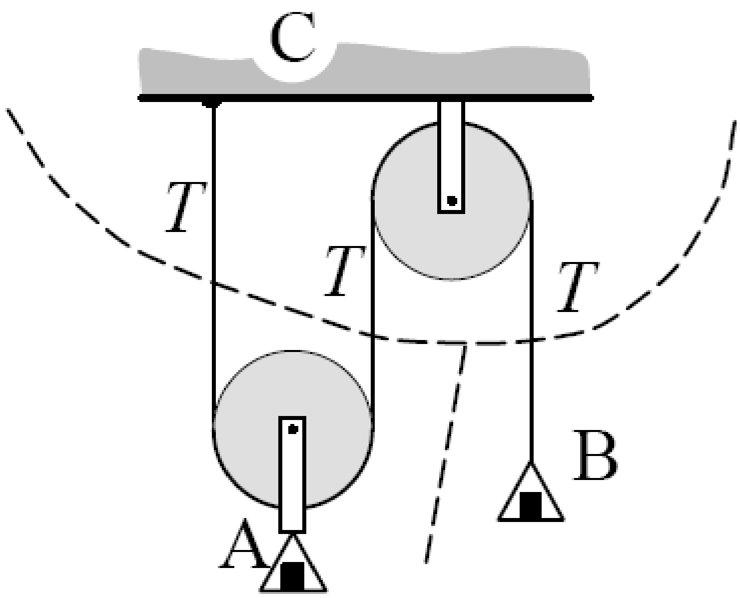
DYNAMICS EXAMPLE 5

Two particles, one of mass 25 grams, the other of mass 50 grams, are connected by a string passing over a frictionless pulley fixed at the apex of a fixed triangular block, so that each is in contact with one of its two smooth sloping faces. If the lighter particle is in contact with a face that makes an angle of 60° with the horizontal, and the apex angle is 90°, find the acceleration of the heavier particle.

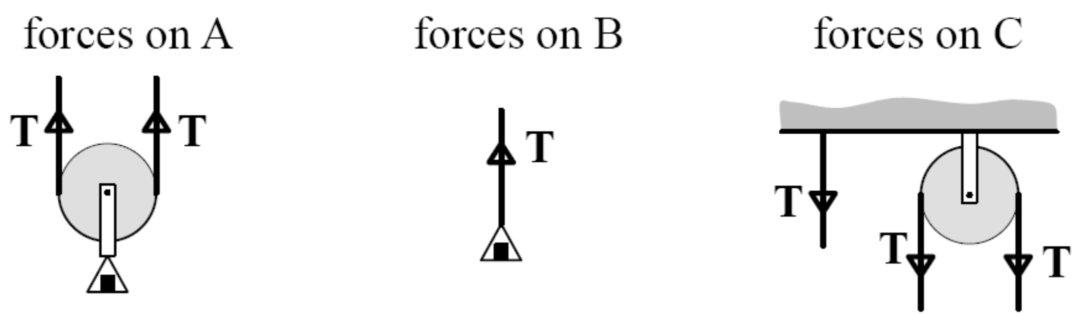
For more complicated systems, you will need to consider the arrangement in order to determine the forces acting. For example:



If the **Tension** in the wire is *T*N, and the pulleys are considered to be weightless and frictionless, we can consider the forces acting on the three parts A, B and C separately (a technique called Free Body Analysis):



Take a section along the dotted line and separate the bodies into Free Bodies



and so:
the force on A is 2*T* N upwards
the force on B is *T* N upwards
the force on C is N
downwards

Similarly, if the acceleration of B is *a* ms^{-2} **downwards**, the acceleration of A will be 1/2*a* ms^{-2} upwards.

WORK

Work is done whenever the **movement** of a force occurs.

The movement does not have to be along the **direction** of the force. What determines the amount of work done is the component of the force in the direction of the displacement, and the magnitude of the displacement (or equivalently, the component of the displacement in the direction of the force and the magnitude of the force).

In fact, the work done is the vector Dot Product of these two quantities. The unit of work is the **Joule** (abbreviation J) which is equivalent to 1 N m.

DYNAMICS EXAMPLE 6

Find the work done by a horse in towing a boat 100 metres along a straight section of canal if the tension in the tow rope, which makes an angle of 15° to the canal bank, has a constant value of 500 Newtons.

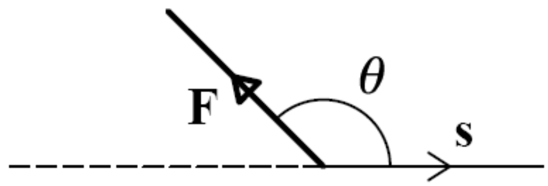
Formally, the work done by a vector force **F** when its point of application is displaced by **s** is the Dot Product of these two vectors, that is:

$$W = \mathbf{F}.\mathbf{s}$$

Notes

1. The definition is the same whether the velocity is zero or constant (equilibrium) or variable. .
2. Work is a **scalar** quantity, but it can be positive or negative. If work is negative, we say the work is **being done on** the force, rather than by it.

3. If work is negative, the direction of the displacement is opposite to the direction of the force, for example:



$$W = F s \cos(\theta)$$

but $\theta > 90°$, so $\cos(\theta) < 0$ and hence W is negative

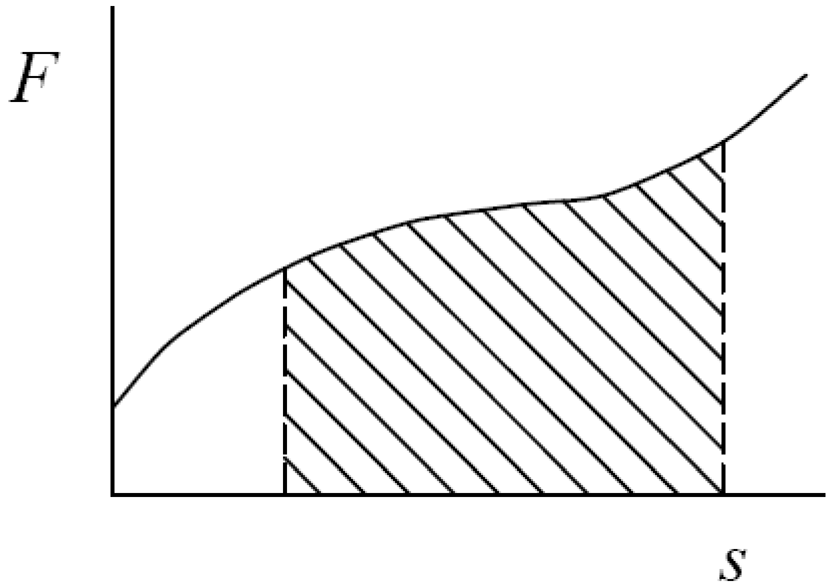
4. Negative work frequently arises when considering friction forces because, by definition, these act in the opposite direction to the motion. Also, work is done against the friction force when an object is in motion.

5. Work is an action of a force, not a constituent of a body. Thus when several forces act on an object, you consider the work done by or against each of them. However, if a body is in equilibrium, the sum of the work done by all the applied forces will be zero.

6. For a **variable** force, work is defined to be:

$$W = \int \mathbf{F} \cdot d\mathbf{s}$$

We shall not be considering such general cases, but we can deal with situations in which the force is changing in value, but is always in the direction of the displacement vector. The work is then evaluated as the area under the curve obtained by plotting force as a **function** of displacement:



DYNAMICS EXAMPLE 7

Find the total work done by the engine of a car of mass 1 tonne in travelling at a constant speed a distance of 1 km up a hill inclined at an angle of arcsin (0.01) to the horizontal if the resistance to motion is 700 N.

ENERGY

Energy is the capacity (potential) to do work. Alternatively, doing work is the CHANGE IN the amount of energy **expended** by something.

Energy is a parameter of a system (object or collection of objects). Like **work** it is measured in joules (J).

There are **two** basic forms of energy in classical mechanics, that is forms of Kinetic and Gravitational Potentail energy.

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Kinetic Energy (KE)

This is energy of motion. Any moving object with mass possesses kinetic energy. It is defined to be equivalent to the work required to accelerate the object from zero velocity to the final **velocity**. An expression for kinetic energy can be derived very simply if we assume the rate of acceleration, and hence the force causing it, is **constant** (if it isn't, we can still derive the expression, but we need calculus to do it).

Suppose an object of mass *m* is **at rest (zero velocity)**. A constant **force** *F* is then applied to it, causing a constant acceleration of *a* , so that at some time later it has been **displaced** from its start point by amount *s* and is travelling with velocity *v* :

$$
\begin{aligned}
&\text{Newton II:} & F &= m\,a \\
&\text{Eqn of motion:} & v^2 &= 2as \quad \text{or} \quad s = \frac{v^2}{2a} \\
&\text{Work done:} & W &= Fs \\
& & &= (ma)\left(\frac{v^2}{2a}\right)
\end{aligned}
$$

Hence the **Kinetic (potential) energy** of an object of mass *m* travelling with speed *v* is:

$$E = \frac{1}{2}mv^2$$

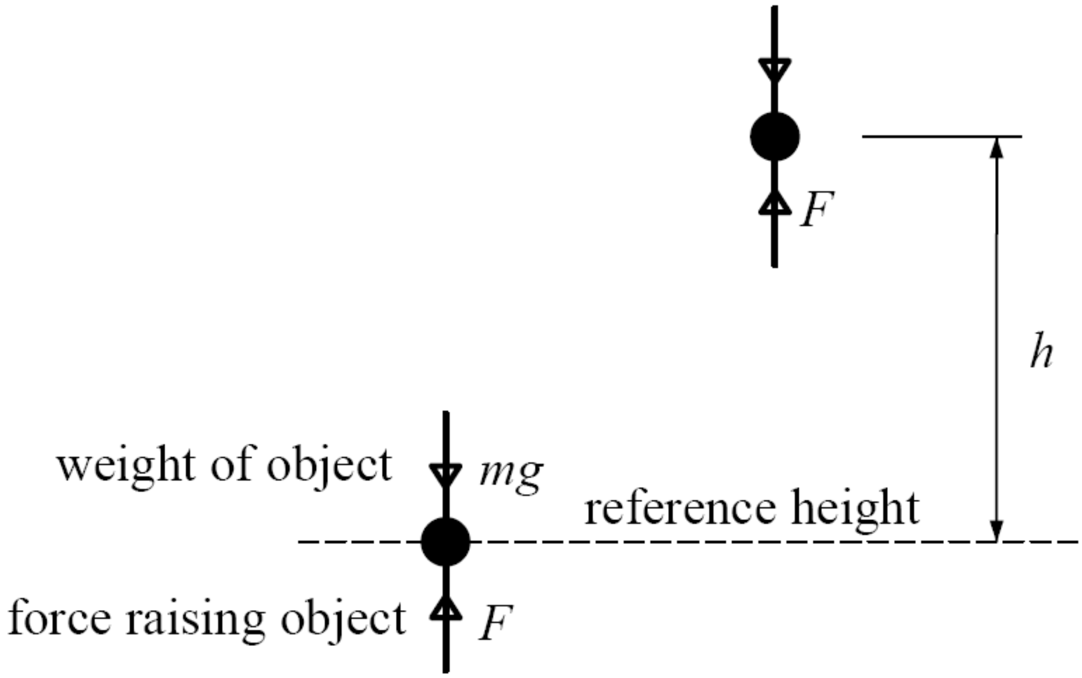
Gravitational Potential Energy (PE)
This is the energy possessed by an object or system due to its position in a gravitational field. It is sometimes said that potential energy is the energy of position (which is where its name comes from).

There many different forms of potential energy depending upon what is causing work to be done when the position or arrangement changes, but most of these forms do not concern us.

The two forms that are important in mechanics at this stage are Kinetic potential energy (KPE) and Gravitational potential energy (GPE).

We shall defer consideration of **Kinectic** potential energy until the next unit. Here, we shall examine Gravitational potential energy

This is the amount of work required to raise an object from some datum or reference position to another height in a gravitational field:



It is assumed that the object is raised (or lowered) at constant velocity, that is the acceleration is **zero** and so the force doing the lifting is equal in magnitude to the weight of the object (*mg*) .

Lifting force:	F	$=$	mg
Distance moved:	s	$=$	h
Work done:	W	$=$	Fs
		$=$	$(mg)(h)$

Hence the change in gravitational potential energy of an object of mass m when it is raised a **distance** h is :

Change in GPE

$$E = mgh$$

where g is the acceleration due to gravity.

Clearly, if an object is **lowered** rather than being raised, h will be **negative**, and the change in GPE will have a negative value (the GPE will **decrease**).

Note It is physically difficult, if not impossible, to define an absolute value of gravitational potential energy. You always calculate a **change** in GPE from the value at some **datum** position. The choice of where to measure from is up to you. In terms of calculations, this is equivalent to assuming the GPE to be zero at a certain height and then treating the value obtained from the *mgh* expression as an addition or subtraction. We shall generally take thisapproach.

Mechanical Energy (ME)

Mechanical energy of an object or system is defined to be the sum of its kinetic and gravitational energies (taking gravitational potential energy to be zero at some chosen height, as mentioned above). There is a very important principle concerned with mechanical energy.

Principle of the Conservation of Mechanical Energy

The **total potential energy** (KE+PE) of a system will **be constant (conserved)** providing:

1. there is no external force, other than the force of gravity, on the system, and
2. energy is not converted to an alternate form such as heat due to friction.

Notes

1. as far as the first condition is concerned, there may be an external force, as long as it does not do work (it could be at right angles to the displacement of the object for example).
2. The total potential energy includes both **kinetic** and gravitational forms, where appropriate.
3. If **Conservation of Energy a**pplies, then it will always be by far the easiest way to solve the problem. Of course, mechanical energy may not be conserved, so you need to check the two conditions stated above. If it isn't then you have to use alternative approaches.

POWER

Power is the amount of work done **per unit time.** Therefore:

$$P = \mathbf{F}.\mathbf{v}$$

Since power is a Scalar quantity, **F** may be **positive or negative.** .

The unit of power is the WATT (abbreviated to W) which is 1 Js^{-1}

DYNAMICS EXAMPLE 8

A locomotive of mass 125 tonnes is pulling a train of mass 200 tonnes along a level section of track. Find the tension in the coupling between the locomotive and the rest of the train if, when the train is travelling at a speed of 180 km h^{-1}, the resistance to motion of the locomotive is 10 kN, the resistance to motion of the rest of the train is 25 kN and the engine of the locomotive is generating a power of 2.56 MW.

EFFICIENCY

When energy is transferred from one form to another (for example kinetic energy to electrical energy) it may not be possible to make **ALL** the energy available in its new form, and some energy may be ' **lost**' in the form of heat.

The efficiency of the conversion is therefore the amount of available (**nett**) energy at the end, **divide** by the amount of energy you started with (usually expressed as a percentage):

$$\% \text{ Efficiency} = \frac{\text{usable energy at end}}{\text{energy you started with}} \times 100$$

Alternatively, as **Power** is the amount of energy per second, this may be given in the form:

$$\% \text{ Efficiency} = \frac{\text{power out}}{\text{power in}} \times 100$$

DYNAMICS EXAMPLE 9

In a hydro–electric installation, water enters a turbine through a pipe of diameter 50 cm at a pressure P of 0.5 MPa. Find the power output of the generator if the turbine/generator set has an overall efficiency of 25% and water flows through the turbine at a rate of 0.25p m^3 per second.

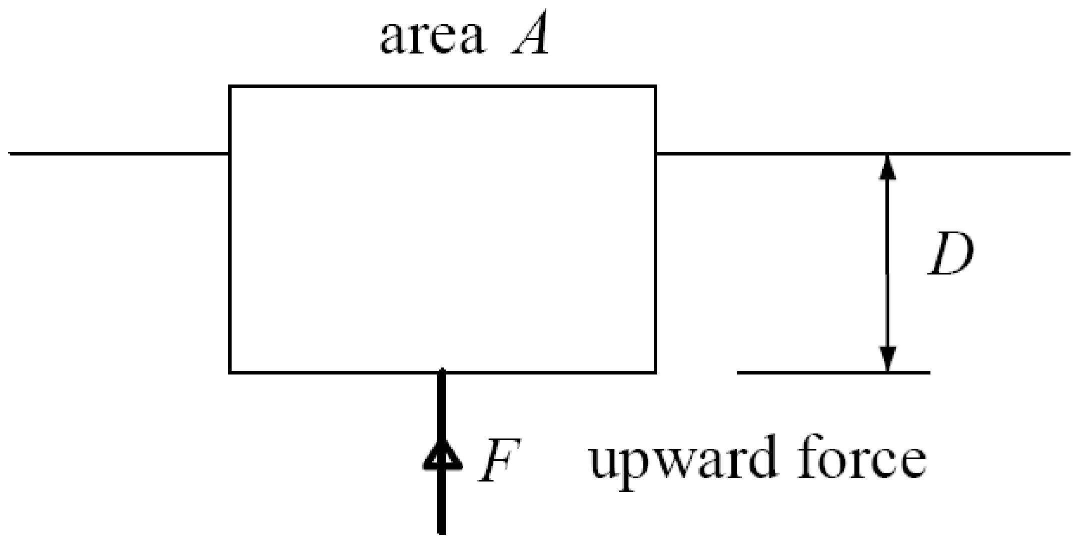
ARCHIMEDES' PRINCIPLE

Archimedes (287–212 BCE). Lived in Syracuse (part of modern Sicily). Killed in the second Punic War.

This has nothing really to do with Dynamics, but the end of this unit is a convenient place to introduce it.

Archimedes' principle states that when an object is immersed in a **fluid**, there is an upward force on the object equal to the mass of fluid displaced (taken up by) the object.

For example, consider a block of cross–sectional area A floating in water (density ρ) with length D of the block below the water level:



Volume of fluid displaced: $V = AD$
Mass of fluid displaced: $M = V\rho$

Weight of fluid displaced: $W = Mg$
Upward force on block: $F = AD\rho g$