

Pedestrian Injuries: Effects of Impact Speed and Contact Stiffness

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ABSTRACT

It is shown that the recently introduced concept of Peak Virtual Power (PVP) as an injury criterion can predict injuries to pedestrians, and that the injury severity is proportional to V^2 , for slight injuries, and V^3 for serious and fatal injuries. Simulations using MADYMO for head and chest injuries show that, to a first degree of approximation, the influence of vehicle contact stiffness on the acceleration of the body is approximately linear. It is therefore concluded that the effects of vehicle contact stiffness and impact speed on injury severity in a collision involving a pedestrian may be approximately modelled as:

$$AIS \propto PI\% \propto PVP \propto K_c \cdot V^n$$

where: AIS = the injury severity of the struck pedestrian

PI% = the probability of injury

PVP= the Peak Virtual Power input to the system

K_c = the normal contact stiffness of the impacting vehicle (N/mm)

V = the vehicle velocity at impact (kph)

n = 2 for slight injuries, and 3 for serious and fatal injuries.

From this simple model it can be deduced that speed reduction is the most important parameter for the reduction of injury severity to pedestrians, as it is a function of V^2 or V^3 , with the attendant advantages that it is immediate, and it also reduces the number of collisions. Reducing vehicle stiffness will have a linear effect on injury severity, and this will only be apparent after a number of years due to the improved vehicles designs filtering into the fleet. A systems-solution will be necessary to achieve a marked reduction in the severity of pedestrian injuries and, because of the timescales involved for vehicle improvements to be seen, then an urgent start should be made on effective countermeasures.

1. INTRODUCTION

Collisions between road vehicles and pedestrians cost the lives of over 6500 people in the European Union each year, with a further 90,000 being seriously injured (IRTAD 1998) [1]. The vast majority of these collisions involve an impact with a passenger car front-end [2], which has led to over thirty years of research to improve pedestrian safety [3-7]. The main factor in reducing the injury severity arising from vehicle-pedestrian impacts is speed [4]. There is a widely perceived need to further reduce injuries through pedestrian friendly car design, and several studies in this field were performed by working groups of EEVC [8-12]. Based on this research a test method has been proposed in Europe to assess the pedestrian protection offered by the front of passenger car [12]. Recently, Yang et.al. [13] have reported part of an on-going programme of research into pedestrian protection, involving an extensive parametric study of the multiplicity of factors involved in pedestrian impacts. Possible benefits were indicated from both the reduction of speed and modifications to the design of vehicle front ends. Although a considerable amount of research has been conducted on the effects of vehicle-pedestrian impacts, it is still not clear what are the proportions of the effects attributable to the major parameters of speed and stiffness. It is the purpose of this paper to try to clarify the relative importance of speed and stiffness, using recent advances in correlations between the physical parameters and injury severity.

2. COST/BENEFITS

There have been several extensive studies to quantify the effect of the introduction of the Proposed European Pedestrian Protection Procedure on the costs and benefits to society. None of these have been universally accepted as being an accurate indication of the outcome of vehicle-pedestrian impacts.

While there are currently no known cost benefit studies for the latest test procedure, the likelihood is that no better assessment could be made than before. As a result of this, five different studies based on the 1994 draft procedure, were analysed [14-18]. These, together with up to date European pedestrian casualties and 1998 UK casualty prevention costs, were used to determine the possible range of benefit [19].

Severity	1999 European Casualties*	1998 UK Cost of Prevention per Casualty (Euros)
Fatal	6,200	1,706,481
Serious	72,000	191,743
Slight	235,000	14,779

Table 1.

* Fatal casualty figures obtained from IRTAD 1999. Serious casualties have been estimated on a rough guide of 10 x Fatales plus 15% for under-reporting (IRTAD) as there are too many definitions to make international comparisons valid. IRTAD have no figure for slight casualties, so a similar proportionality between UK numbers fatal and slight casualties was used.

Using these benefits and the latest European casualty data, ranges of possible benefits can be calculated. If it is assumed that the proposal becomes legislation in the year 2002, and a five year compliance period is instituted, it is necessary to estimate the proportion of pedestrian friendly cars in relation to the European vehicle fleet. Since only new models produced after the implementation date must conform to the test requirements, there will be a relatively low percentage at the onset, which will increase in future years. The process of replacing the

entire European fleet could take more than 15 years to complete given the average vehicle life, and the age of a vehicle involved in a fatal vehicle-pedestrian collision in the West Midlands shows an almost linear trend, see Fig.1.

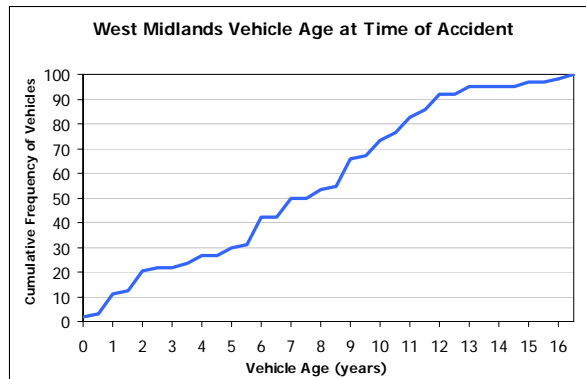


Fig.1.

As of 2002, no current car model conforms to the proposed legislation, which means that any potential savings will not be realised until the procedure takes effect, which is likely to be after 2007. The overall benefit will be greatly affected by several trends seen across Europe. These included an ever increasing vehicle fleet, a European wide reduction in pedestrian casualties and the future costs of preventing casualties. The problem arises in quantifying these trends to predict the situation through until 2022. Benefits per annum were found to range from 8.2M Euros to 44.4M Euros after the first year and between 917M Euros and 5.5 billion Euros for a fully compliant vehicle fleet. The range of average benefit was calculated based on studies [15, 16] and [17], which appeared to be in reasonable agreement with each other on casualty reductions. From this, Europe can expect an annual benefit to range between 16M Euros after the first year and 1.9 billion Euros when all vehicles comply with legislation.

3. THEORETICAL CONSIDERATIONS:

Biomechanical injuries result in the straining or separation (fracture, shearing, tearing or rupture) of biological tissues. The tissues themselves, as opposed to the structures (organs) in which they are arranged, generally have a high Bulk Modulus, as they are either water or Collagen based [20-22]. Soft biological tissues can be modelled as "nearly incompressible" visco-elastic-plastic materials [22, 23], and bone as elastic-plastic [20-22, 24]. In [25] it is considered that injuries in Impact Trauma may be viewed as "mechanical dissipative processes" i.e. they require an expenditure of work. The derivation of Peak Virtual Power as an injury criterion is reproduced from [25] in Appendix 1 for completeness. The derivation in Appendix 1 is a damage model, and it is necessary to map injuries on to damage. Here it is postulated that the severity of injuries in Impact Trauma are proportional to the energy input to the system. For a given impact, over a given time interval, the amount of damage (severity) will be proportional to the rate of damage production times the time interval, a "rate", "dosage", or "exposure" criterion then becomes:

$$\text{Severity of Injury} \propto D \propto (\dot{D})\Delta t \propto (\sigma : \dot{\epsilon}^p)\Delta t \propto PVP \quad (1)$$

Consider the impact of a deformable body travelling at velocity V , impacting a deformable structure with a ride-down of S over a time Δt , at a constant deceleration, as shown in Fig.1, and equate the external power to the internal power. This crash pulse is applicable to an ideally “fully-coupled” vehicle-pedestrian impact (soft contact), as the crash pulse experienced by both car and pedestrian are the same.

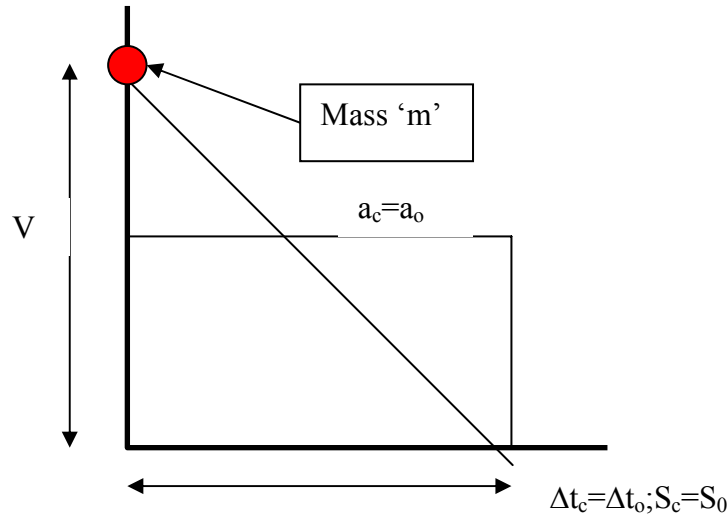


Fig.2. Crash pulse for a fully-coupled pedestrian

Subscript ‘c’ relates to the vehicle, subscript ‘o’ relates to the pedestrian.

For this particular impact, the impact velocity ‘ V ’ is also the change in velocity in the impact ΔV .

$$\text{Max / Peak Power} \quad \frac{\partial \hat{U}}{\partial t} = F \cdot \hat{V} = F \cdot V \quad (2)$$

$$\text{Then PVP is} \quad \frac{1}{m} \left(\frac{\partial \hat{U}}{\partial t} \right) = a \cdot V \quad (3)$$

$$\text{But } V = a \cdot \Delta t, \text{ then} \quad \frac{1}{m} \left(\frac{\partial \hat{U}}{\partial t} \right) = a^2 \Delta t = \hat{a} = \frac{V^2}{\Delta t} \quad (4)$$

To render this measure specific to the vehicle it is necessary to normalise on the vehicle ride-down. This expresses the power per unit mass for the occupant as a function of the severity of collision of the vehicle. Normalising on the vehicle ride-down for this pulse, $S_c = \frac{V}{2} \Delta t$, where S_c is the coupled ride down, then:

$$PVP = \frac{S_c}{m} \left(\frac{\partial \hat{U}}{\partial t} \right) \propto V^3 \approx \Delta V^3 \text{ or } (ETS)^3 \quad (5)$$

Severity of injury is generally measured using the “Abbreviated Injury Scale” (AIS) [26], which is an “ordinal” or integer non-linear increasing scale, derived from empirical data, and expresses the probability of death from the injury sustained. Therefore, if Peak Virtual Power is a valid measure of injury severity, then it should be proportional to the maximum AIS score for the body regions. Furthermore, it was reported by Evans [27] that the probability of injury is given by:

$$PI\% = \left(\frac{\Delta V}{\alpha} \right)^n \quad (6)$$

Therefore for this crash pulse it is assumed that:

$$AIS \propto PI\% \propto PVP \propto aV|_{\max} \propto \hat{a} \propto a^2 \Delta t \propto V^3 \propto \Delta V^3 \text{ or } (ETS)^3 \quad (7)$$

which should apply for fully coupled pedestrians. It should be emphasised here that any of the alternative forms of PVP shown in equation 7 are admissible as injury criteria.

In [28] it is also shown for “de-coupled” impacts, when in equation 4 $S_{dc} = C \cdot \Delta t$, where S_{dc} is the de-coupled ride down and ‘C’ is a constant, then:

$$AIS \propto PI\% \propto PVP \propto aV|_{\max} \propto \hat{a} \propto \bar{a}^2 \Delta t \propto V^2 \propto \Delta V^2 \text{ or } (ETS)^2 \quad (8)$$

That the indicators of severity of injury are either proportional to V^2 or V^3 for de-coupled or coupled impacts respectively is of considerable importance as the V^2 is the “Upper Bound” or worst case, whereas the V^3 is the “Lower Bound”. This is counter-intuitive, and arises as the coordinates on an AIS versus ΔV plot of (0,0) and (AIS=6, ΔV) are fixed, and as the constant of proportionality is higher for the square relationship this gives higher AIS values at lower ΔV 's for the square relationship than for the cubic.

Consider the impact of two bodies interspersed with linear elastic springs as in CRASH3 [29].

Let: M_c = mass of car M_b = mass of human body
 K_c = normal contact stiffness of car K_b = normal contact stiffness of body
 $\beta = \frac{K_b}{K_c}$

The equation of motion for the body mass, after CRASH3 [29] is:

$$M_b \ddot{x}_b = \left[\frac{K_c K_b}{K_c + K_b} \right] (x_c - x_b) \quad (9)$$

Where x_c = displacement of the car
 x_b = displacement of body part

therefore
$$\ddot{x}_b = \left[\frac{\beta M_b}{(x_c - x_b)(1 + \beta)} \right] K_c \propto K_c \quad (10)$$

but
$$PVP \propto a^2 \Delta t \propto a \propto \ddot{x}_b \propto K_c \quad (11)$$

therefore in general combining 7, 8 and 11 gives:

$$PVP \propto AIS \propto PI\% \propto K_c(a) \propto K_c(a^2 \Delta t) \propto K_c(\Delta V^n) \quad (12)$$

Currently there are no studies available which cross correlate PVP or AIS with pedestrian impacts, this is attempted here by way of both simulations and comparison with the real-world.

4. SIMULATIONS:

Simulations of head and thorax impact are chosen, as these are the most important body parts in pedestrian impact [2]. In Fig.3. a comparison is shown between the results of HIC computed from a MADYMO simulation of a Hybrid III head [30] of stiffness of 6500 N/mm, being the approximate stiffness of a 50th percentile male as indicated in [30]. The head model is impacted with a 8kg impactor consisting of a simple plane of a mass approximately twice the mass of the body part [30], and PVP as ($a^2 \Delta t$) normalised at a stiffness of 300 N/mm, the form of results are again seen to be very close.

It can be seen that the relationship between PVP/HIC and stiffness is approximately linear over the typical range of stiffness encountered for the front ends of vehicles i.e. 100N/mm<stiffness<600N/mm unless chassis parts are encountered. The head impacts considered above all occur with $\beta \gg 1$, and are therefore short duration (hard or bounce) impacts. It is interesting to note that in the “Extended Prasad/Mertz Curves” being considered by NHTSA [31] it is proposed to replace HIC₃₆ with HIC₁₅ for such impacts, to recognize the short duration.

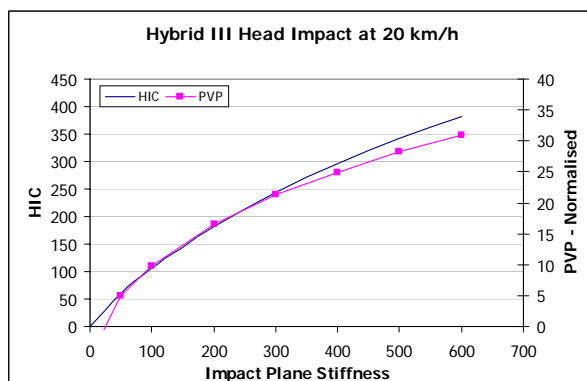


Fig.3.

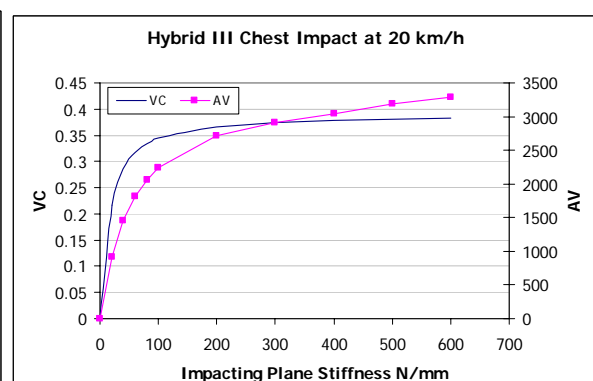


Fig.4.

Considering the Thorax, then from equation 7 as $\sigma \propto \varepsilon \propto \Delta \propto \hat{\Delta}$ (where Δ is chest compression) then $\sigma : \dot{\varepsilon}^p \propto \hat{\Delta} \hat{V}$, which is of the same form as the Viscous Criterion. To compare PVP with the Viscous Criterion then the $aV|_{\max}$ form of equation 7 was calculated from a MADYMO simulation of the impact of a HYBRID III thorax with a stiffness of 20 N/mm representing the 50th percentile male ($\beta \ll 1$). A 20kg impactor, again of an effective mass approximately twice the mass of the body part was taken, and $VC|_{\max}$ was calculated using the appropriate MADYMO routine [30], the output for $aV|_{\max}$ was very much larger than for $VC|_{\max}$ and so the results for $aV|_{\max}$ were normalised on $VC|_{\max}$; the results are shown in Fig.4. The MADYMO implementation of the sternum compression is highly non-linear and, the MADYMO model [30] starts to limit the amount of chest compression after a chest compression of 80mm [30]. From the agreement obtained above, and the mathematical form of the relationships, it is concluded that the Viscous Criterion is of a form predicted by Peak Virtual Power, hence cross correlating with [28], where it is shown that there is a linear relationship between AIS and PVP, then:

$$PVP \propto AIS \propto PI\% \propto K_c \quad (13)$$

It should be noted that K_c is the normal stiffness of the contact plane. In reality the majority of vehicle-pedestrian impacts will involve non-normal impacts (glancing blows) of varying severity, and so equation 13 represents the worst case scenario. In practice the “shape-factor” of the vehicle front end may be very influential.

The variations of HIC, VCmax and PVP are also explored for variations in impact velocity, and the results are shown in Figs. 5 and 6. The values of the parameters used are the same as quoted earlier. For the head impact with $\beta \gg 1$ the agreement between HIC and PVP is close, and V^2 is the better model, reflecting the hard or bounce type de-coupled impact. For the thorax impact simulations $\beta \ll 1$, and so these are “soft” or coupled impacts and would be expected to conform to a V^3 which from Fig.6 is seen to be the case.

From the simulations conducted it can then be generally concluded that as PVP is proportional to both HIC and VCmax, and as PVP can be mapped onto AIS, then as a hard impact ($\beta \gg 1$) can be modelled by $PVP \propto V^2$, and a soft impact ($\beta \ll 1$) as $PVP \propto V^3$, then the form of equations 7 and 8 have been confirmed.

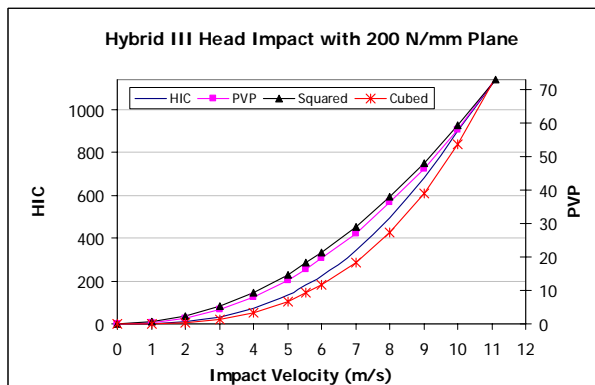


Fig.5.

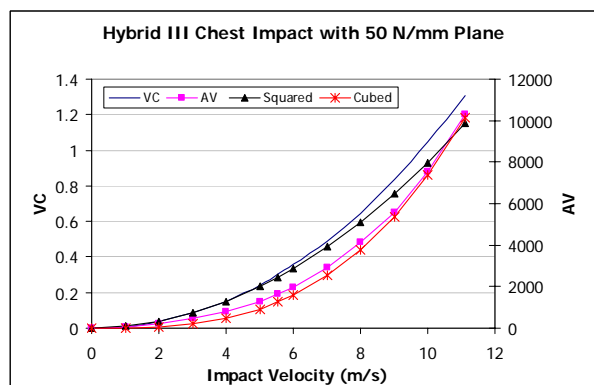


Fig.6.

5. REAL-WORLD EFFECTS:

The effects of speed on the probability of injury in pedestrian impacts in the real world have been known since Ashton and Mackay's study in 1979 [32], shown in Fig7. Also plotted in Fig.7 are the correlations with PVP expressed both as the coupled (V^3) and the de-coupled (V^2) models. It can be seen that the correlation with V^3 is superior for the serious and fatal injuries whereas the correlation with the V^2 model is better for the slight injuries. This leads to the tentative conclusion that the pedestrian is closely coupled to the vehicle during the injury phase of the contact, for serious and fatal injuries, and less closely coupled i.e. glancing blows, for the slight injuries. However, much more in-depth work would need to be done before this hypothesis could be considered proven.

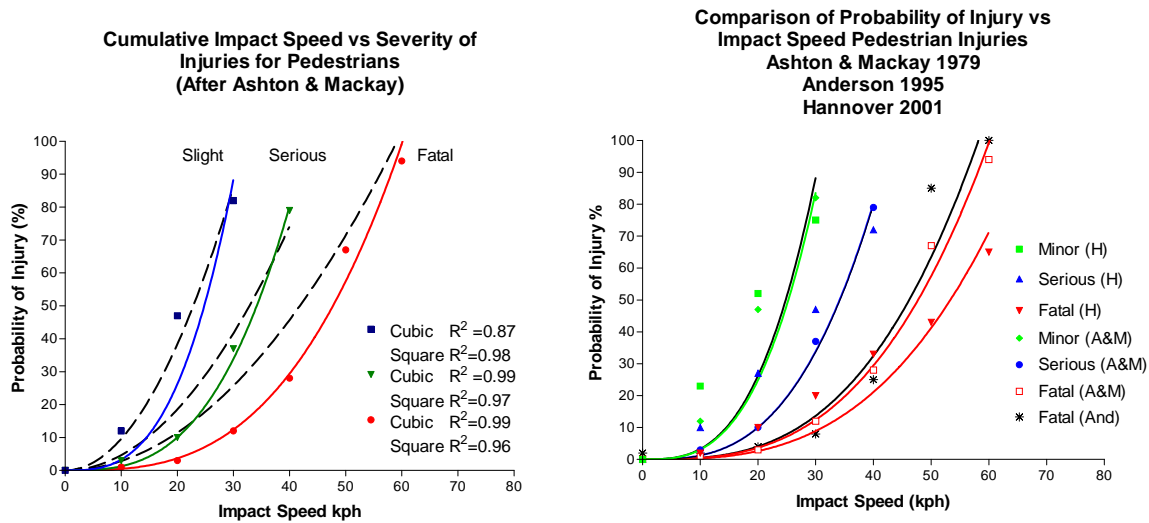


Fig.7.

Fig.8.

It should be noted that equation 13 with $n=2$ apparently corresponds to Limpert's equation [33]. However, this correspondence is superficial, as here a proportionality is stated, and K_c is the vehicle normal contact stiffness and not an empirical constant.

In Fig.8 more recent studies [34, 35] are also plotted on the same axes as Fig.7. It is immediately evident that little has changed since 1979, with the possible exception that the spread of the results from the recent Hannover study appear larger than the other two studies, and the fatalities appear to occur at slightly higher impact speeds than previously recorded.

Therefore, simulations using MADYMO for head and chest injuries have shown that, to a first degree of approximation, the influence of vehicle contact stiffness on the acceleration of the body is close to being linear. Real-world pedestrian studies have shown that slight injuries are proportional to V^2 and serious and fatal injuries proportional to V^3 , it is therefore concluded that in general the effects of vehicle contact stiffness and impact speed on injury severity in a collision involving a pedestrian may be approximately modelled as:

$$AIS \propto PI\% \propto PVP \propto K_c.V^n \quad (14)$$

where: AIS = the injury severity of the struck pedestrian
 PI% = the probability of injury
 PVP= the Peak Virtual Power input to the system
 K_c = the normal contact stiffness of the impacting vehicle (N/mm)
 V = the vehicle velocity at impact (kph)
 n = 2 for slight injuries, 3 for fatal injuries.

From this simple model it can be clearly seen that, of the two parameters considered, speed reduction is the most important parameter for the reduction of injury severity to pedestrians, as it is a function of V^2 or V^3 . This strategy comes with the attendant advantages that (i) it is immediate, (ii) it also reduces the number of collisions. It has been shown that there is a linear relationship between a change in mean speed and the number of accidents [36]. Reducing vehicle stiffness will have a linear effect on injury severity, and this will only be apparent after a number of years due to the improved vehicles designs filtering into the fleet. To achieve a marked reduction in pedestrian injuries will require a full systems solution of: forward looking anti-collision sensors coupled to automatic brake actuation; external air-wall technology; softer front ends; and optimum front end shapes; most of these technologies have already been demonstrated [37].

As part of its 10 Year Plan for Transport the Government has set a target of a 40% reduction in those killed or seriously injured in road accidents; a 50% fall in children killed or seriously injured; and a 10% cut in minor casualties. Within the timescales for these targets, the contribution from a reduction in pedestrian injuries will largely be dependent on speed reduction.

6. CONCLUSIONS:

It is concluded that the effects of vehicle contact stiffness and impact speed on injury severity in a vehicle-pedestrian collision may be approximately modelled as:

$$AIS \propto PI\% \propto PVP \propto K_c.V^n$$

From this simple model it can be deduced that speed reduction is the most important parameter for the reduction of injury severity to pedestrians, as it is a function of V^2 or V^3 , with the attendant advantages that it is immediate, and it also reduces the number of collisions. Reducing vehicle stiffness will have a linear effect on injury severity, and this will only be apparent after a number of years due to the improved vehicles designs filtering into the fleet. A systems solution will be necessary to achieve a marked reduction in the severity of pedestrian injuries, and because of the timescales involved for vehicle improvements to be seen, then an urgent start should be made on effective countermeasures.

ACKNOWLEDGEMENTS

The Authors would like to thank the West Midlands Police Crash Investigation Unit, and the Birmingham and Solihull Coroners for all their help with the real-world cases. Thanks are

also due to the Engineering and Physical Sciences Research Council for the provision of Studentships to Mr. G. Coley and Mr. P. de Oliveira.

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APPENDIX 1: Derivation of Peak Virtual Power as an Injury Criterion

The concept of virtual power has been used extensively to accurately formulate various theorems in the mechanics of materials, and has recently been applied to Impact [38]. If the process is chemically and electrically neutral, the Clausius-Duhem Inequality for small deformation of incompressible bodies is [38]:

$$\sigma : \dot{\varepsilon} - \rho(\dot{f} + s\dot{T}) - \frac{1}{T} q \cdot \nabla T \geq 0 \quad (\text{A1})$$

where: σ = Stress Tensor, $\dot{\varepsilon}$ = Total Strain Rate Tensor, ρ = Mass Density, f = Helmholtz Free Energy, s = Entropy, T = Absolute Temperature, q = Heat Flux Vector

This is a Tensor equation as it must be invariant to transformations. Proceeding according to [39] assuming that the total strain tensor can be decomposed into the elastic and plastic (reversible and irreversible) strain tensors as:

$$\varepsilon = \varepsilon^e + \varepsilon^p \quad (\text{A2})$$

This decomposition applies to both elastic-plastic and visco-plastic materials [40]. The appropriate form of the Helmholtz Free Energy (the constitutive relationships) [41] may be described as a function of the following variables:

$$f = f(\varepsilon^e, D, T) \quad (\text{A3})$$

where ε^e = the elastic strain tensor
 D = the "Damage Tensor"

Now differentiating (A3) and substituting in (A1) gives the specific rate of entropy production ($\dot{\Phi}$) as:

$$\rho\dot{\Phi} = \sigma : \dot{\varepsilon}^p - \rho \left(\frac{\partial f}{\partial D} \right) \dot{D} - \frac{1}{T} q \cdot \nabla T \geq 0 \quad (\text{A4})$$

For a mechanical system, assuming the contribution of heat flux can be decoupled [42],

$$\sigma : \dot{\varepsilon}^p \geq \rho \left(\frac{\partial f}{\partial D} \right) \dot{D} \quad \text{or} \quad \sigma : \dot{\varepsilon}^p \propto \dot{D} \quad (\text{A5})$$

All rational injury criteria must either be equal to, or proportional to, equation A5 otherwise the second law of thermodynamics will be violated.